Ribando

WRITING PROOFS

When you write a mathematical proof, your purpose should be to provide a convincing argument of the assertion. The easier you make it for the reader to read your proof, the easier he/she will be convinced. For this reason it is important that you write clearly, concisely, and in a style consistent with the standards of mathematical writing.

Prof. Munkres of MIT is well known for his high standards of mathematical writing. The widespread use of his texts Topology and Analysis on Manifolds is testimony to his clear mathematical writing. I frequently refer to these texts when deciding how to format or write clearly some mathematical prose. Here are some of his tips for how you can write proper mathematics.

Comments on Style
James Munkres

1. Write in complete sentences.
2. Punctuate! (Correctly, if possible.)
3. Write legibly. (An illegible proof is incorrect by definition.)
4. Avoid the "stream of consciousness" style popularized by Wm. Faulkner. When you finish a thought, stop, put down a period, and take a good breath before you begin the next sentence (with a capital letter, please).
5. Steer a clear middle ground between too much detail and not enough. Give reasons for your answer sufficient to convince your reader (in this case, the grader) that your argument is correct and you know why it is correct. But don't fill the pages by checking each tiny detail in writing; it only bores the reader and gives you writer's cramp.

   At one extreme of style are those sparsely-written texts (such as Rudin) that require the reader to ponder each sentence and fill in most of the details. At the other extreme are those solutions written by your most conscientious fellow-student, so full of details that the basic idea is invisible!

   Try to hit somewhere in the middle.
6. Don't use mathematical symbols as parts of speech in an ordinary sentence. Bad examples:
   (a) Consider the set of all numbers < 1.
   (b) Consider the $\cap$ of the sets A and B.
   (c) Consider the function f mapping $A \longrightarrow B$.

Here is how to write these sentences correctly:

(a) Consider the set of all numbers less than 1.
or
Consider the set of all numbers x such that $x < 1$.

(b) Consider the intersection of the sets A and B.
or
Consider the set $A \cap B$.

(c) Consider a function f mapping A into B.
or
Consider a function $f : A \to B$.

7. Don't use logical symbols <u>at all</u>. The symbols $\exists, \ni, \forall, \exists!, \vee, \wedge$, as well as the abbreviations s.t., w.r. to, are to be avoided in mathematical writing. In papers in <u>logic</u>, these symbols constitute a part of the subject matter and are completely appropriate. In formal mathematical discourse, on blackboard or paper, they are often "parts of speech," in a sort of mathematical shorthand. However, they are not allowed by editors in formal mathematical writing.

   Just as you wouldn't submit a history paper that is written partly in secretarial shorthand, don't submit a math paper written in mathematical shorthand!

8. One exception to the rule in (7) is the use of symbols $\Longrightarrow$ (implies) and $\Longleftarrow$ (is implied by) and $\Longleftrightarrow$ (is equivalent to). One of course does not use these symbols as word-substitutes, any more than one uses $<$ or $+$ or $\cup$ as word-substitutes. But usage is allowed in phrases such as: "We show that $(a) \Longrightarrow (b) \Longrightarrow (c)$," or "To show (a) and (b) are equivalent, it suffices to show that $(a) \Longrightarrow (b)$ and $(b) \Longrightarrow (a)$."

   There is a reason why editors (at least those who are mathematicians) enforce rule (7) strictly. Most mathematical readers find sentences in which this rule is violated quite unreadable, just as they find secretarial shorthand unreadable. They translate the sentence into English language (or German, or French, or. . .) mentally, before attempting to understand it.

   Occasionally a textbook editor (who is usually not a mathematician) will let an author get away with violating these rules. Here is a horrendous example, quoted from a well-know text, <u>verbatim</u>:

$$\text{Let } f : [0, \Omega) \longrightarrow [0, \Omega) \text{ be s.t. } f(\alpha) < \alpha \text{ for all } \alpha \geq \text{some } \alpha_0.$$
$$\text{Then } \exists\ \beta_0\ \forall\ \beta\ \exists!\ \alpha \geq \beta : f(x) \leq \beta_0.$$

   Some people can grasp the meaning of these two sentences immediately; most mathematicians can't!

   (P.S. In this passage, $[0, \Omega)$ denotes what we call S_Ω. Can you rewrite this passage understandably?)