

Midterm
Math 6C
Professor A. Johnson
April 11, 2002

Instructions: This is a closed book exam. No calculators are allowed. Show all work and justify all answers.

15 points

1. You have a business in which you have a choice as to how many hours you work each day, (x), and how much you spend on operating costs in dollars per day, (y). Your production, as a function f of x and y is shown below:

$y \backslash x$	6	6.5	7	7.5
20	971	1010	1098	1110
21	990	1029	1086	1132
22	1009	1049	1107	1153

a) What does $f(7, 21)$ represent in this scenario?

b) Estimate $f_x(7, 21)$.

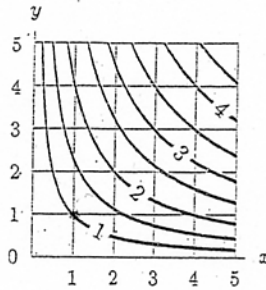
c) What does $f_x(7, 21)$ represent in this scenario?

2

15 points

2. Draw some level curves for the function $z = y^2 - x^2$. Include the level curve that goes through the point $(1, 1)$.

b) In the picture shown below, draw in the gradient of the function at the point $(1, 1)$.



c) Using ~~your~~^{the} picture above, is the directional derivative of the function at the point $(1, 1)$ in the direction of $\vec{i} + 2\vec{j}$ positive or negative? Explain.

3. Find an equation for the tangent plane to $z^2 - 2x^2 - 2y^2 = 12$ at the point $(1, -1, 4)$. Use your tangent plane to estimate the value of z at $(x, y) = (1.5, 0.5)$.

12 points

22 points

4. A hot metal plate is situated on an xy -plane such that the temperature $T(x, y) = k \frac{1}{\sqrt{x^2 + y^2}}$. If the temperature at $(3, 7)$ is 10, find k .

b) Find the rate of change of T at $(2, 5)$ in the direction of $\vec{i} + \vec{j}$.

c) In what direction does T decrease the most rapidly?

d) In what direction is the rate of change 0?

12 points

5. For this problem, we will consider a tree trunk to have the shape of a right circular cylinder. First, write down the volume of the tree trunk as a function of the radius r and the height h .

b) Suppose the diameter of the trunk increases 1 inch per year and the height of the trunk increases 6 inches per year. How fast is the volume of the tree trunk increasing when it is 10 ft high and 14 inches in diameter?

6. Mark the following problems True or False.

A) Let $P = f(m, d)$ be the purchase price in dollars of a used car with m miles on its engine and with original cost d dollars when new. Then $\frac{\partial P}{\partial m}$ and $\frac{\partial P}{\partial d}$ have the same sign.

B) If $f(x, y)$ is a function with the property that $f_x(x, y)$ and $f_y(x, y)$ are both constant, then f is linear.

C) If $f(x, y)$ has $f_x(a, b) = 0$ and $f_y(a, b) = 0$, then f is constant everywhere.

D) For every function $f(x, y)$, we have $f_{xy} = f_{yx}$.

E) If $f(x, y)$ is a function of 2 variables defined for every x and y , then $f(0, y)$ is a function of 1 variable.

F) The plane $x + 2y - 3z = 1$ passes through the origin.

G) Two contours of $f(x, y)$ with different heights never intersect.

H) The level surfaces of $g(x, y, z) = x + 2y + z$ are parallel planes.