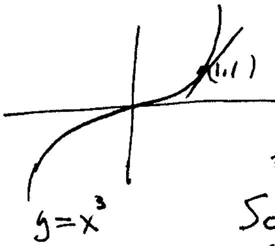
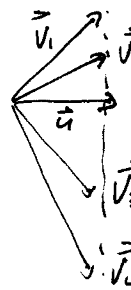
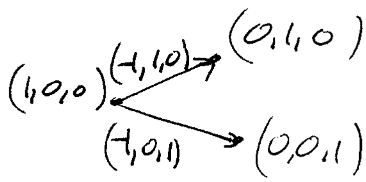
① a) 

i) The slope of the tangent line is $f'(x) = 3x^2$. At $(1,1)$, the slope is $3\cdot 1^2 = 3$. So if (a,b) is a vector parallel to the graph at $(1,1)$, then $b=3a$. So any vector of the form $(a,3a)$ is parallel. To make it a unit vector, we need to have $1 = \|(a,3a)\| = \sqrt{a^2+9a^2} = \sqrt{10}\,|a|$, or $|a| = \frac{1}{\sqrt{10}}$. So $\left(\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right)$ & $\left(\frac{-1}{\sqrt{10}}, \frac{-3}{\sqrt{10}}\right)$ are possible answers.

ii) A vector $\perp$ to (a,b) is $(-b,a)$, because $(a,b)\cdot(-b,a) = 0$. So $\left(\frac{-3}{\sqrt{10}}, \frac{1}{\sqrt{10}}\right)$ or $\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$ will work.

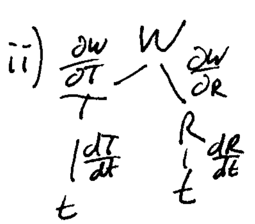
b) No. 

Here, $\vec{v}_1\cdot\vec{u} = \vec{v}_2\cdot u = \vec{v}_3\cdot u = \vec{v}_4\cdot u$.

c) 

A normal vector to the plane is $(-1,1,0)\times(-1,0,1) = (1,1,1)$. So an equation for the plane is
$1(x-1) + 1(y-0) + 1(z-0) = 0$, or $x+y+z=1$

② a) i) $\frac{\partial W}{\partial T} < 0$ means that if temp. goes up, wheat production goes down.
$\frac{\partial W}{\partial R} > 0$ means that if rainfall goes up, wheat production goes up.

ii) 

$$\frac{dW}{dt} = \frac{\partial W}{\partial T}\cdot\frac{dT}{dt} + \frac{\partial W}{\partial R}\cdot\frac{dR}{dt}$$
$$\approx -2\cdot 0.15 + 8\cdot -0.1 = -1.1$$

2 b) i) We want $f_{\vec{v}} = \nabla f(3,4,5) \cdot \frac{\vec{v}}{\|\vec{v}\|}$

$$\nabla f = (10x - 3y + yz,\ -3x + xz,\ xy)$$

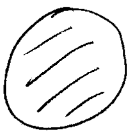
So $\nabla f(3,4,5) = (38, 6, 12)$.

$\|\vec{v}\| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{3}$, so $\frac{\vec{v}}{\|\vec{v}\|} = \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}$

So $f_{\vec{v}} = (38,6,12) \cdot \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right) = \frac{32}{\sqrt{3}}$

ii) In the direction of $\nabla f(3,4,5) = (38,6,12)$

iii) $\|\nabla f(3,4,5)\| = \|(38,6,12)\| = \sqrt{38^2 + 6^2 + 12^2}$

3) a)  $x^2+y^2 \le 1$

1st, Find interior crit. pts. of f

$\nabla f = (2x, 4y)$ - only crit pt. is $(0,0)$.

2nd, find possible extrema on the bdy. $g(x,y) = x^2 + y^2 = 1$

$\nabla g = (2x, 2y)$ - never 0 on $g = 1$.

So solve $\nabla f = \lambda \nabla g$, or $(2x, 4y) = \lambda(2x, 2y)$

$2x = 2\lambda x$, so $\lambda = 1$ or $x = 0$

If $\lambda = 1$, $4y = 2y$, so $y = 0$. Since $x^2+y^2=1$, $x^2 = 1$, so $x = \pm 1$. So $(\pm 1, 0)$ are possibilities.

If $x = 0$, then $y^2 = 1$, so $y = \pm 1$. So $(0, \pm 1)$ are possibilities.

Check values of f at $(0,0)$, $(\pm 1, 0)$, & $(0, \pm 1)$.

$f(0,0) = 0$ ← min. value

$f(\pm 1, 0) = 1$

$f(0, \pm 1) = 2$ ← max. value

3 b) Tangent plane is $z = 5 + f_x(3,4)(x-3) + f_y(3,4)(y-4)$

$f_x = \frac{\partial}{\partial x}\left((x^2+y^2)^{\frac{1}{2}}\right) = \frac{x}{\sqrt{x^2+y^2}}$; $f_y = \frac{y}{\sqrt{x^2+y^2}}$

$f_x(3,4) = \frac{3}{5}$, $f_y(3,4) = \frac{4}{5}$

So the plane is $z = 5 + \frac{3}{5}(x-3) + \frac{4}{5}(y-4)$

4 a) R could be two regions:
$$\begin{array}{c} 0 \le z \le e^{-x} \\ 0 \le y \le 2 \\ -\infty < x \le 1 \end{array} \quad \text{OR} \quad \begin{array}{c} 0 \le z \le e^{-x} \\ 0 \le y \le 2 \\ 1 \le x < \infty \end{array}$$

The 1st region has infinite volume, so let's assume that the question meant to ask about the second region. In that case, the answer is

$$\int_1^\infty \int_0^2 e^{-x}\,dy\,dx = \int_1^\infty \left(ye^{-x}\Big|_{y=0}^{2} \right) dx$$

$$= \int_1^\infty 2e^{-x}\,dx$$

$$= \lim_{a\to\infty} \int_1^a 2e^{-x}\,dx$$

$$= \lim_{a\to\infty} -2e^{-x}\Big|_1^a$$

$$= \lim_{a\to\infty} \left(-2e^{-a} + 2e^{-1}\right)$$

$$= 0 + 2e^{-1} = \frac{2}{e}.$$