

FINAL EXAM

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1. (a) Find a unit vector
 - (i) parallel to, and
 - (ii) normal tothe graph of the function $f(x) = x^3$ at the point $(1, 1)$.

- (b) If $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w}$ and $\mathbf{u} \neq \mathbf{0}$, does it necessarily follow that $\mathbf{v} = \mathbf{w}$? Explain.

- (c) Find the equation of the plane through the points $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$.

2. (a) A model for wheat production in a given year, W , depends upon the average temperature T and the annual rainfall R . Scientists estimate that the average temperature is rising at a rate of $0.15^\circ\text{C}/\text{yr}$ and that the rainfall is decreasing at a rate of $0.1 \text{ cm}/\text{yr}$. They also estimate that, at current production levels, $\frac{\partial W}{\partial T} = -2$ and $\frac{\partial W}{\partial R} = 8$.
 - (i) What is the significance of the signs of these partial derivatives?

 - (ii) Estimate the current rate of change of wheat production, $\frac{dW}{dt}$.

- (b) Suppose that over a certain region of space the electrical potential V is given by
$$V(x, y, z) = 5x^2 - 3xy + xyz$$
 - (i) Find the rate of change of the potential V at the point $(3, 4, 5)$ in the direction of the vector $\hat{\mathbf{v}} = \mathbf{i} + \mathbf{j} - \mathbf{k}$.

 - (ii) In what direction does V change the most rapidly at the point $(3, 4, 5)$?

 - (iii) What is the maximum rate of change at the point $(3, 4, 5)$?

3. (a) Find the extreme values of $f(x, y) = x^2 + 2y^2$ on the disk $x^2 + y^2 \leq 1$.

- (b) Find an equation for the tangent plane to the surface defined by $z = \sqrt{x^2 + y^2}$ at the point $(3, 4, 5)$.

4. (a) Find the volume of the solid region R bounded by the surface $f(x, y) = e^{-x}$ and the planes $y = 0$, $z = 0$, $y = 2$, and $x = 1$.