1/2/03 Worksheet Solutions — Math 6B

1. Use the ratio test: $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{|x^{n+1}|}{|n+1|} \cdot \frac{|n|}{|x^n|}$
(assuming $x \neq 0$)

$$= \lim_{n\to\infty} \frac{n}{n+1} \cdot |x| = |x|$$

This clearly converges (to 0) if $x=0$, so the ratio test says that it converges if $|x|<1$ & diverges if $|x|>1$. If $x=1$, then we get the harmonic series, which diverges. (If $x=-1$, we get the alternating harmonic series, which c[onverges].) So the series converges for $-1 \le x < 1$, and diverges for $x < -1$ & $x \ge 1$.

2. To have enough to give a scholarship in 1 year, she needs to deposit $\$D_1$ now, where $D_1 e^{0.07} = 1000$, or $D_1 = 1000\, e^{-0.07}$. To give one in two years as well, she must now deposit an additional $\$D_2$, where $D_2\, e^{2 \cdot 0.07} = 1000$, or $D_2 = 1000\,(e^{-0.07})^2$. So to give one every year, she must now deposit $D_1 + D_2 + D_3 + \cdots$

$$= 1000\, e^{-0.07} + 1000\,(e^{-0.07})^2 + 1000\,(e^{-0.07})^3$$

$$= 1000\, e^{-0.07}\left(1 + e^{-0.07} + (e^{-0.07})^2 + \cdots\right)$$

$$= 1000\, e^{-0.07} \cdot \frac{1}{1-e^{-0.07}} \approx 13{,}791.50$$

3. ~~a)~~ These are alt. series, so $|S_n - S| \le a_{n+1}$

a) error $\le \frac{1}{n+1}$, so need $\frac{1}{n+1} < \frac{1}{10}$, or $n = 10$.
The last term added, a_{10}, is positive, so we get an overapproximation.

3) Here, error $\leq \frac{1}{(n+1)!}$, so solve $\frac{1}{(n+1)!} \leq \frac{1}{10}$

$1! = 1$, $2! = 2$, $3! = 6$, & $4! = 24$, so take $n=3$.

This time, the last term added is $\frac{-1}{3!}$, so we get an underestimate.

4) a) No — the terms aren't decreasing.

b) $S_1 = 2$, $S_2 = 1$, $S_3 = 2$, $S_4 = 1 + \frac{1}{2}$, $S_5 = 1 + \frac{1}{2} + 2/3$, $S_6 = 1 + \frac{1}{2} + 1/3$

In general, $S_{2n} = 1 + \frac{1}{2} + \frac{1}{3} + \dots \frac{1}{n}$. These are the partial sums of the harmonic series, so it diverges.

5) a) – diverges (n^{th} term test) b) diverges – use limit comparison to $\sum \frac{1}{n}$

c) converges – limit comparison to $\sum \frac{1}{e^n}$

d) diverges – limit comparison to $\sum \frac{1}{n}$

e) converges – ratio test, or compare to $\sum e^{-k}$

f) converges – limit comparison to $\sum \frac{1}{n^2}$

g) diverges – n^{th} term test