4/25/03 Math 6B WS Solns

(1) $f(x) = \sin 2x + \cos 3x$, so $f(0) = 1$

a) $f'(x) = 2\cos 2x - 3\sin 3x$, so $f'(0) = 2$

$f''(x) = -4\sin 2x - 9\cos 3x$, so $f''(0) = -9$

$f'''(x) = -8\cos 2x + 27\sin 3x$, so $f'''(0) = -8$

So $P_3(x) = 1 + 2x - \frac{9}{2}x^2 - \frac{8}{3!}x^3$

b) $|E_3(x)| \le \frac{M|x^4|}{4!}$, where $|f^{(4)}| \le M$ on $[0, 0.1]$.

Here, $f^{(4)}(x) = \cancel{8\cos 2x}\ 16\sin 2x + 81\cos 3x$, which can never have absolute value bigger than $16 + 81 = 97$. So we can use $M = 97$.

So, $|E_3(0.1)| \le 97 \cdot (0.1)^4/4! \approx 0.000404$

(2) speed at 0 = $s'(0) = 20$, accel. at 0 = $s''(0) = 2$.

So $s(t) \approx P_2(t) = s(0) + s'(0)t + s''(0)\frac{t^2}{2!}$

$= 0 + 20t + t^2$, so $s(1) \approx P_2(1) = 20 + 1 = 21$.

Can't estimate error, since we don't know the third derivative. Probably shouldn't use this to predict where the car will be in one minute, since it could easily have sped up or braked in that time.

(3) See p. 444 of Hughes-Hallett

(4) a) Diverges by limit comparison to $\sum \frac{1}{n}$

b) Converges, by direct comparison of absolute value to $\sum \frac{1}{e^n}$ (p-series), or by alternating series test.

c) Converges, by root test

d) Converges, by ratio test