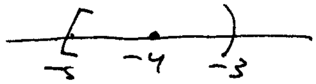
Math 6B Practice Midterm Solns

1. a) diverges - limit comparison with $\sum \frac{1}{n}$
   b) converges - compare with $\sum_{n=0}^{\infty} \frac{1}{3^n}$ ← geometric
   c) converges - ratio test
   d) diverges - n^{th} term doesn't go to 0
   e) converges - ratio test
   f) converges, to $\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{n-1}{2n+1} = \frac{1}{2}$
   g) converges - limit comparison with $\sum a_n$

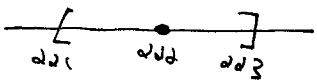
2. $a \cdot \frac{1}{1-r}$. In the notes, or pp. 407-8 of Hughes-Hallett.

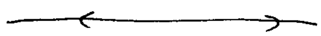
3. Use the integral test. Ex. 1, p 545 of Gillett.

4. $\sum_{n=1}^{\infty} \frac{1}{n^3} = 1 + \frac{1}{8} + \frac{1}{27} + \cdots$ is clearly bigger than 1.
   Also, it is smaller than $\sum_{n=1}^{\infty} \frac{1}{n^2} = \pi^2/6$

5. a) $R=1$, int. of conv. is $[-5,-3)$. 

   b) $R=0$, int. of conv. is $\{0\}$ 

   c) $R=1$, int. of conv. is $[221, 223]$ 

   d) $R=\infty$, int. of conv is $(-\infty,\infty)$ 

6. a) Can do it directly, or: $e^t = 1 + t + \frac{t^2}{2!} + \cdots$, so
   $e^{x^2} = 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \cdots$ (plug in $t = x^2$).

   b) The coeff. of x^{10} is $f^{(10)}(0)/10!$, so $f^{(10)}(0)/10! = \frac{1}{5!}$,
   or $f^{(10)}(0) = \frac{10!}{5!} = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6$

(7) Ratio test gives $R=1$. At $x=1$, the series is $\sum \frac{(-1)^n \cdot 1}{2n+1}$, which converges by the alt. series test. At $x=-1$, the series is $\sum \frac{(-1)^n \cdot (-1)}{2n+1}$, which is just the negative of the 1st series, so it also converges. So the int. of conv. is $[-1, 1]$.

(8) a) $300 + 300(.57) + 300(.57)^2 + \dots + 300(.57)^{24}$
$= 300\left(1 + (.57) + \dots + (.57)^{24}\right) = 300 \, \frac{1-(.57)^{25}}{1-.57}$

b) As $n \to \infty$, the amt. in the bloodstream goes to $\sum_{n=0}^{\infty} 300 \cdot (.57)^n = 300 \cdot \frac{1}{1-.57}$
≈ 697.7, so no, you never quite get to 700 mg.

(9) a) $1-1+1-1+1-\dots$ b) $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots$ c) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\dots$
d) can't happen: If $\sum a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.
e) $1+\frac{1}{2}+1/4+\frac{1}{8}+\dots$ (lots of examples)

(10) $\sin x = x - x^3/3! + x^5/5! - \dots$, so
$\sin 1 = 1 - 1/3! + 1/5! - 1/7! + \dots$

(11) a) $T_4(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!}$

b) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$. If $|x| \le 1$, this is a decreasing alt. series, so the error in $T_4(x)$ is less than the next term,
$\left|\frac{-x^6}{6!}\right| = \left|\frac{-1}{6!}\right| = 1/6!$