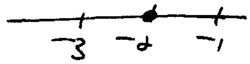
Math 6B Practice Final Solns

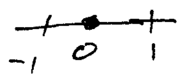
1. a) Use $R = \lim_{n\to\infty}\left|\frac{c_n}{c_{n+1}}\right| = \lim_{n\to\infty}\frac{\frac{1}{n^2}}{\frac{1}{(n+1)^2}} = \lim_{n\to\infty}\frac{n^2+2n+1}{n^2} = 1$. So the radius of conv. is 1. Check endpts:



If $x=-1$, series is $\sum \frac{1}{n^2}$ - convergent p-series.

If $x=-3$, series is $\sum \frac{(-1)^n}{n^2}$ - convergent alt. series. So the interval of convergence is $[-3,-1]$

b) Again, use $R = \lim_{n\to\infty}\left|\frac{c_n}{c_{n+1}}\right| = \lim_{n\to\infty}\frac{n}{n+1} = 1$. So $R=1$; check endpts



If $x=1$, series is $\sum n$ - divergent by n^{th} term test

If $x=-1$, series is $\sum(-1)^n n$ - div. by n^{th}-term test

So the interval of convergence is $(-1,1)$.

c) Since the series is $(x-7)^2/4 + (x-7)^4/16 + \cdots$, c_m is 0 for m odd, so $R = \lim_{n\to\infty}\left|\frac{c_m}{c_{m+1}}\right|$ doesn't exist. Apply the Ratio Test to the power series!

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(x-7)^{2n+2}}{4^{n+1}} \Big/ \frac{(x-7)^{2n}}{4^n}\right| = \lim_{n\to\infty}\frac{(x-7)^2}{4}.$$ The series

converges if this limit is <1, ie, $(x-7)^2<4$, ie, $|x-7|<2$, and diverges if " " " >1, ie, $(x-7)^2>4$, ie, $|x-7|>2$. So the radius of conv. is 2. Check the endpts:

If $x=5$, the series is $\sum \frac{(-2)^{2n}}{4^n} = \sum 1$, diverges.

If $x=9$, " " " $\sum \frac{(2)^{2n}}{4^n} = \sum 1$, diverges

So the interval of convergence is $(5,9)$.

(2) a) Converges by the root test.

b) Converges by the ratio test.

c) $\sum \ln\left(\frac{n}{2n+1}\right)$ diverges by n^{th}-term test

d) Converges by limit comparison with p-series $\sum \frac{1}{n^{3/2}}$

e) Converges - Throw out the first 3 terms and you have a decreasing alternating series.

(3) The series converges by the alt. series test. We know that $|S - S_n| \le a_{n+1}$, so we need $\frac{1}{(n+1)^2} < \frac{1}{10{,}000}$, or $(n+1)^2 > 10{,}000$, or $n+1 > 100$, or $n = 100$ terms.

(4) $y(x) = c_0 + c_1 x + c_2 x^2 + \cdots$. Since $y(0) = 2 = c_0 + c_1 \cdot 0 + c_2 \cdot 0 + \cdots c_0$,

$y(x) = 2 + c_1 x + c_2 x^2 + \cdots$

Now, $y' + y = 1$

$c_1 + 2c_2 x + 3c_3 x^2 + \cdots + 2 + c_1 x + c_2 x^2 + \cdots = 1$

So: $c_1 + 2 = 1 \Rightarrow c_1 = -1$; $2c_2 + c_1 = 0 \Rightarrow c_2 = \frac{1}{2}$; $3c_3 + c_2 = 0 \Rightarrow c_3 = \frac{-1}{3 \cdot 2}$

... Get $y = 2 - x + \frac{x^2}{2} - \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots$

(5) One soln: $\frac{dP}{dt} = P(P-L)$, where L is the minimum # of rhinos necessary for them to find each other in order to mate.

(6) a) $P' = 0$ when $P = 0, 50,$ or 200

b) $P' > 0$ for $50 < P < 200$

c) $P' < 0$ for $0 < 50 < P$ & $P > 200$.

d) If you start w/ 150, in the long run they'll increase to 200.