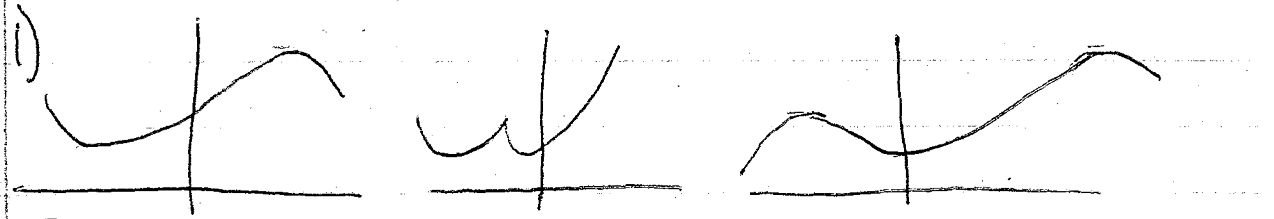
9/13/02 Derivatives Worksheet Solutions

1)



Every other solution is a vertical translation of the above.

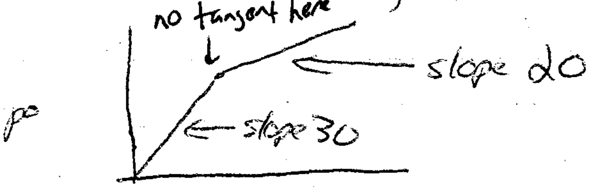
2) Reaches max. ht. when derivative goes from + to − (rising to falling). $f'(t) = -4t + 8$, which goes from + to − at $\underline{t = 2}$. Its height then is $f(2) = -2(2)^2 + 8 \cdot 2 = 8$

3) $\frac{dV}{dp}$ is rate of change of vol. with respect to pressure. Roughly, it's how much the vol. will change if we increase the pressure by 1 atm.
dA/dr is rate of change of area w.r.t. radius. Roughly, it's how much the area will change if we increase the radius by 1 unit.

4) $\frac{d}{dr}\left(\frac{Gm_1m_2}{r^2}\right) = \frac{d}{dr}\left(Gm_1m_2r^{-2}\right) = -2Gm_1m_2r^{-3} = \frac{-2Gm_1m_2}{r^3}$.
It's negative, which makes sense because the further apart the bodies are (ie, the bigger r is), the less gravitational attraction b/w them.

5) Let D be the total distance traveled (in miles). So if she went the first half of that at 30 mph, it took her $\frac{\text{dist}}{\text{rate}} = \frac{D/2}{30} = D/60$ hrs.
Similarly, it took her $\frac{D/2}{20} = D/40$ hrs to cover the second half.
Thus, overall she went D miles in $D/60 + D/40 = \frac{5D}{120} = \frac{D}{24}$ hrs.
So her average speed ~~has~~ was $\frac{\text{dist}}{\text{time}} = \frac{D}{D/24} = 24$ mph.

The position fn is not differentiable, because she goes from 30 to 20 mph instantly:



6) $\frac{d}{d\theta}(\sin\theta) = \lim_{h\to 0} \frac{\sin(\theta+h) - \sin\theta}{h}$

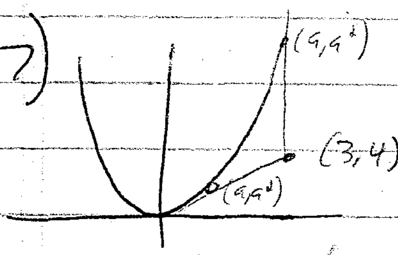
$= \lim_{h\to 0} \frac{\sin\theta\cos h + \cos\theta\sin h - \sin\theta}{h}$

$= \lim_{h\to 0} \frac{\sin\theta(\cos h - 1) + \cos\theta\sin h}{h}$

$= \lim_{h\to 0} \frac{\sin\theta(\cos h - 1)}{h} + \lim_{h\to 0} \frac{\cos\theta \sin h}{h}$

$= \sin\theta\left(\lim_{h\to 0}\frac{\cos h - 1}{h}\right) + \cos\theta\left(\lim_{h\to 0}\frac{\sin h}{h}\right)$

$= \sin\theta \cdot 0 + \cos\theta \cdot 1 = \boxed{\cos\theta}$

7) 

If the line from $(3,4)$ to (a,a^2) is tangent to the parabola at (a,a^2), then the slope of that line is equal to the slope of the tangent line at a, namely $f'(a)$.

Since $\frac{d}{dx}(x^2) = 2x$, we have:

$\left(\text{slope of line from } (3,4) \text{ to } (a,a^2) \rightarrow\right) \frac{4-a^2}{3-a} = 2a \;(\leftarrow \text{slope of tangent line at } (a,a^2))$

$4 - a^2 = 6a - 2a^2$

$a^2 - 6a + 4 = 0$, or $a = 3 \pm \sqrt{5}$

The equation of the line from $(3,4)$ with slope $2a$ is $y - 4 = 2a(x-3)$, so the two lines are

$y - 4 = 2(3+\sqrt{5})(x-3)$ & $y - 4 = 2(3-\sqrt{5})(x-3)$