WISEMAN **MATH 5** **10/25/01**

LINEAR APPROXIMATION AND NEWTON'S METHOD WORKSHEET

1. Use linear approximation to estimate $\sin 43°$.

Convert to radians! $43° = \frac{43 \cdot \pi}{180}$

$f(x) \approx f(a) + f'(a)\cdot(x-a)$. Here, take $a = 45° = \pi/4$, &

$\sin'(x) = \cos x$. So:

$$\sin\left(\frac{43\pi}{180}\right) \approx \sin(\pi/4) + \cos(\pi/4)\cdot\left(\frac{-2\pi}{180}\right)$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \frac{-2\pi}{180}$$

$$\approx .6824$$

(Really, $\sin 43° \approx .6820$)

2. a) The equatorial radius of the earth is approximately 3960 miles. Suppose that a wire is wrapped tightly around the earth at the equator. Approximately how much must this wire be lengthened if it is to be strung all the way around the earth on poles 1 inch above the ground? (Use linear approximation!) What's the exact answer?

length $= 2\pi \cdot$ radius, or $L = 2\pi r$

$r = 3960$. One inch poles means $\Delta r = 1$ in.

$$\text{So } \Delta L \approx \frac{d}{dr}(2\pi r)\Big|_{r=3960} \cdot \Delta r$$

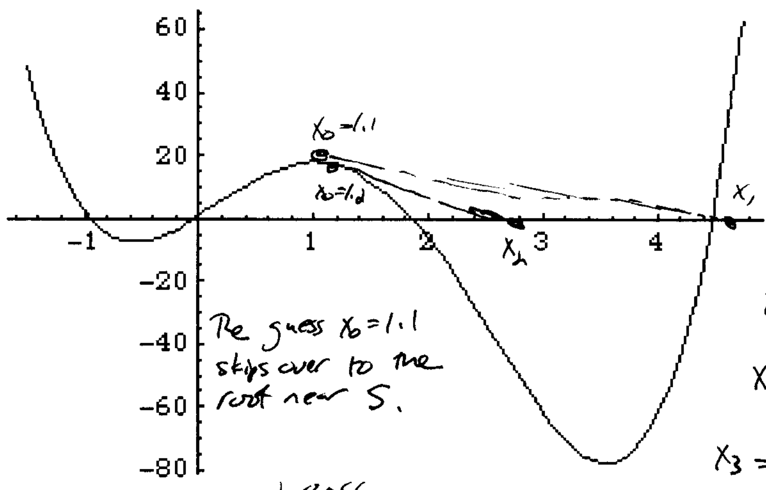
$$= 2\pi \cdot 1 = 2\pi$$

This is exactly right! L is a linear function of r, so linear approximation gives exactly the right answer.

b) The radius of a basketball is approximately 6 inches. Suppose that a wire is wrapped tightly around the basketball at its equator. How much must this wire be lengthened if it is to be strung all the way around the basketball on poles 1 inch above its surface?

Again, $\Delta L = 2\pi \cdot 1$.

3. A portion of the graph of the polynomial $3x^4 - 16x^3 + 6x^2 + 24x + 1$ is given below. It looks like it's got a root slightly smaller than 2. Using Newton's method, try to find that root, starting with initial guess $x_0 = 1.1$. Explain what happens. Try $x_0 = 1.2$ now. What happens?



The guess $x_0 = 1.1$ skips over to the root near 5.

$f' = 12x^3 - 48x^2 + 12x + 24$

$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

x_1, $x_0 = 1.1$

$x_1 = 1.1 - \dfrac{17.7563}{-4.908} = 4.7178$

$x_2 = 4.7178 - \dfrac{53.8746}{272.3421} = 4.5200$

$x_3 = 4.5200 - \dfrac{6.7396}{205.7257} = 4.4872$

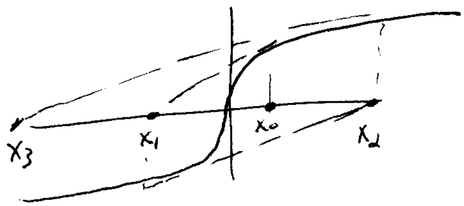
OR: $x_0 = 1.2$, $x_1 = 2.904$, $x_2 = 1.8056$, $x_3 = 1.8453$

4. Use Newton's method to try to find a root of the function $f(x) = \sqrt[3]{x}$, using $x_0 = 1$ as your initial guess. Explain what happens (a picture might be helpful).

$f'(x) = \frac{1}{3} x^{-2/3}$

$x_{n+1} = x_n - \dfrac{x_n^{1/3}}{\frac{1}{3} x^{-2/3}} = x_n - 3x_n = -2x_n$

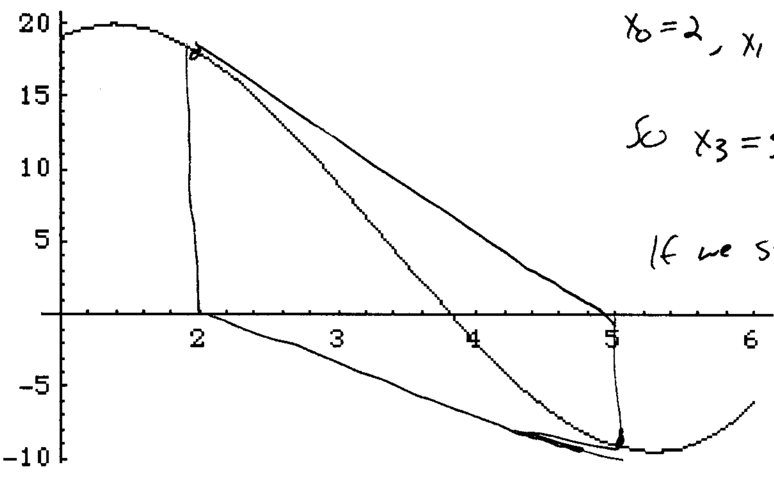
$x_0 = 1$, $x_1 = -2$, $x_2 = 4$, $x_3 = -8$, ...



It just keeps getting farther & farther from 0.

5. A portion of the graph of the polynomial $x^3 - 10x^2 + 22x + 6$ is shown below. Using Newton's method, try to find the root near $x = 4$, using $x_0 = 2$ as your initial guess. What happens? What would happen if you took $x_0 = 5$?

$f'(x) = 3x^2 - 20x + 22$



$x_0 = 2$, $x_1 = 2 - \dfrac{18}{-6} = 5$, $x_2 = 5 - \dfrac{-9}{-3} = 2$

So $x_3 = 5$, $x_4 = 2$, $x_5 = 5$, $x_6 = 2$, ...

If we started at $x_0 = 5$, we'd get

$x_0 = 5, 2, 5, 2, 5, 2$ ...