Math 5 Calculus Wiseman 9/27/02

Trig Functions and Chain Rule Worksheet

(1) Calculate the derivatives of the following functions: $\sin e^x$, $\cot\theta$, $\sec\theta$, $\tan^2 x$, $\tan x^2$

$(\sin e^x)' = (\cos e^x)e^x$

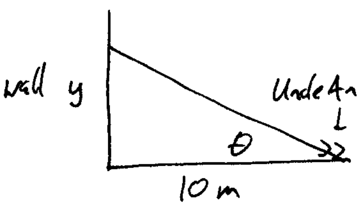
$(\cot\theta)' = \left(\frac{\cos\theta}{\sin\theta}\right)' = \frac{\sin\theta(-\sin\theta) - \cos\theta(\cos\theta)}{\sin^2\theta} = \frac{-\sin^2\theta - \cos^2\theta}{\sin^2\theta} = \frac{-1}{\sin^2\theta} = -\csc^2\theta$

$(\sec\theta)' = \left(\frac{1}{\cos\theta}\right)' = \frac{-(-\sin\theta)}{\cos^2\theta} = \frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta} = \tan\theta\sec\theta$

$(\tan^2 x)' = 2\cdot\tan x\cdot\sec^2 x$

$(\tan x^2)' = (\sec^2 x^2)\cdot 2x$

(2) Uncle Ant is shining a laser beam on a wall. If the wall is 10 meters away, and the angle θ that the beam makes with the ground is increasing at a rate of 0.2 radians/second, how fast is the height y of the spot on the wall increasing?



$\frac{y(t)}{10} = \tan\theta(t)$, so $y(t) = 10\tan\theta(t)$

$y'(t) = 10\sec^2\theta(t)\cdot\theta'(t)$

Since $\theta'(t) = 0.2$, $y'(t) = 2\sec^2\theta(t)$

(3) (a) The number B of grams of bacteria living on the leftover eggplant in my fridge after t days is given by the function $B(t) = 1000e^{0.1t}$. How fast is the amount of bacteria growing after ten days?

$B'(t) = (1000e^{0.1t})' = 1000e^{0.1t}\cdot 0.1 = 100e^{0.1t}$

So $B'(10) = 100e^{0.1\times 10} = 100e \approx 272$ g/day

(b) The tastiness T of the eggplant is a function of the number of bacteria living on it: $T = 1/B$. How fast is the tastiness decreasing after ten days? What are the units?

$T(t) = \frac{1}{B(t)}$, so $T'(t) = \frac{-B'(t)}{(B(t))^2}$. Thus $T'(10) = \frac{-100e}{(1000e)^2} = \frac{-1}{10{,}000e}$

$\approx \frac{-1}{27{,}200}$ $\frac{\text{metric yums}}{\text{day}}$

(4) A cubical block of ice with edges 20 in. long begins to melt at 8 am. Each edge decreases at a constant rate thereafter and each is 8 in. long at 4 pm. What was the rate of change of the block's volume at noon? (Is this problem realistic physically?)

s = edge length V = volume $V = s^3$, so $V' = 3s^2s'$. s' is constant.

After 8 hours, s has decreased by 12, so $s' = \frac{-12}{8} = -3/2$. So at noon,

$s = 20 - \frac{3}{2}\cdot 4$ (4 hrs. from 8 to noon), or $s = 14$.

Thus $V'(\text{noon}) = 3s^2(\text{noon})\,s'(\text{noon}) = 3\cdot 14^2\cdot -3/2 = -882$ in^3/hr

(Realistically, s' probably wouldn't be constant. The rate of melting is probably proportional to surface area.)