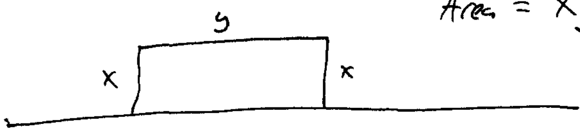
Math 5 Calculus Wiseman 11/4/02

Optimization Worksheet

(1) A farmer has 200 yd of fence with which to construct three sides of a rectangular pen; an existing long, straight wall will form the fourth side. What dimensions will maximize the area of the pen?



Area $= xy$, $2x + y = 200$, so $y = 200 - 2x$, so

area $= f(x) = x(200 - 2x) = 200x - 2x^2$

Endpts are $x = 0$ & $x = 100$ (so $y = 0$).

Find crit pts: $f'(x) = 200 - 4x$. Always exists, so set to 0: $0 = 200 - 4x$, $x = 50$

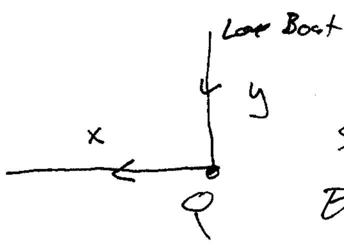
$f(0) = 0$

$f(50) = 50 \cdot 100 = 5000$

$f(100) = 0$

So the max dim are $x = 50$, $y = 100$

(2) At noon the Love Boat is 50 mi north of the good ship Lollipop and is steaming south at 16 mi/h. The good ship Lollipop is headed west at 12 mi/h. At what time are they closest together, and what is the minimal distance between them?



Minimize distance squared: $d^2 = x^2 + y^2$

Let t = # hrs since noon. Then $x = 12t$ and $y = 50 - 16t$.

So minimize $f(t) = (12t)^2 + (50 - 16t)^2 = 400t^2 - 1600t + 2500$

Endpts: $t = 0$ or "∞"

Find crit. pts.: $f'(t) = 800t - 1600$. Always exists, so only crit pt is $t = 2$

$f(0) = 50^2 = 2500$

$f(2) = (24)^2 + (18)^2 = 900$

$\lim_{t \to \infty} f(t) = \infty$

So the min is at $t = 2$, ie, 2 pm. The distance then is $\sqrt{d^2} = \sqrt{900} = 30$ mi.

(3) The Leopard Automotive Co. estimates that they can sell 5000 cars a month at \$40,000 each, and that they can sell 500 more cars per month for each \$1000 decrease in price.

(a) What price per car will bring the largest gross income?

(b) If each car costs \$16,000 to make, what price will bring the largest total profit?

a) Income is N (# sold) $\times$ P (price). Let d = # of \$1000 decreases in price. Then $N = 5000 + 500d$ & $P = 40{,}000 - 1000d$, so income $I(d) = NP = -500{,}000d^2 + 15{,}000{,}000d + 200{,}000{,}000$

$I'(d) = -1{,}000{,}000d + 15{,}000{,}000$, so crit. pt. is $d = 15$. Endpts. are $d = -\infty$ & $d = 40$ (why?)

$I(40) = 0$, $I(15) = 312{,}500{,}000$, $\lim_{d \to -\infty} I(d) = -\infty$. So max income is when $d = 15$, or price $= \$25{,}000$.

b) profit $\pi(d) = I(d) - 16{,}000N$. Crit pts: $0 = \pi'(d) = I'(d) - 16{,}000N' = -1{,}000{,}000d + 15{,}000{,}000 - 8{,}000{,}000$

or $0 = -1{,}000{,}000d + 7{,}000{,}000$, or $d = 7$. $\pi(40) = -640{,}000$, $\pi(7) = 144{,}500{,}000$.

Max profit is when $d = 7$, or price is \$33,000.