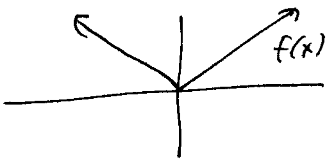
(1) (12 pts) For each of the following, draw the graph of a function f with the stated properties, or explain why no such function can exist.

(a) f is differentiable but not continuous.

This can't happen - every diff. fn. must be continuous

(b) f is continuous but not differentiable.

$f(x) = |x|$

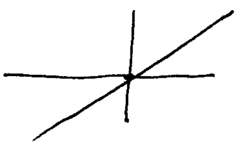


Not diff. at $x=0$.

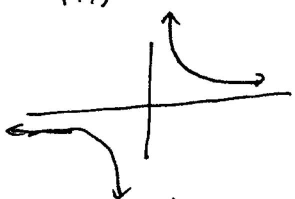
(LOTS OF POSSIBLE ANSWERS)

(c) f is differentiable, but $\dfrac{1}{f}$ is not.

$f(x) = x$



$\dfrac{1}{f(x)} = \dfrac{1}{x}$

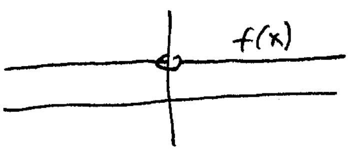


$\left(\dfrac{1}{f(x)}\right)' = \left(\dfrac{1}{x}\right)' = \dfrac{-1}{x^2}$, which doesn't exist at $x=0$.

(LOTS OF POSSIBLE ANSWERS)

(d) $\lim_{x\to 0^-} f(x) = 1$, $\lim_{x\to 0^+} f(x) = 1$, but f is not continuous at $x = 0$.

$f(x) = \dfrac{x}{x}$



- Does not exist at $x=0$

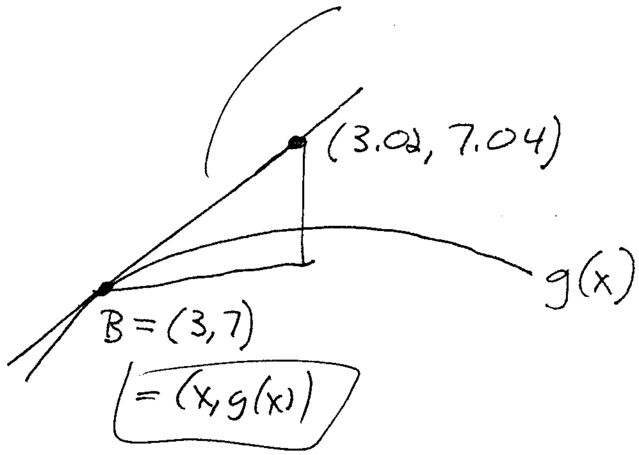
(LOTS OF POSSIBLE ANSWERS)

(2) (10 pts) Use the figure below to fill in the blanks in the following statements about the function g at point B.

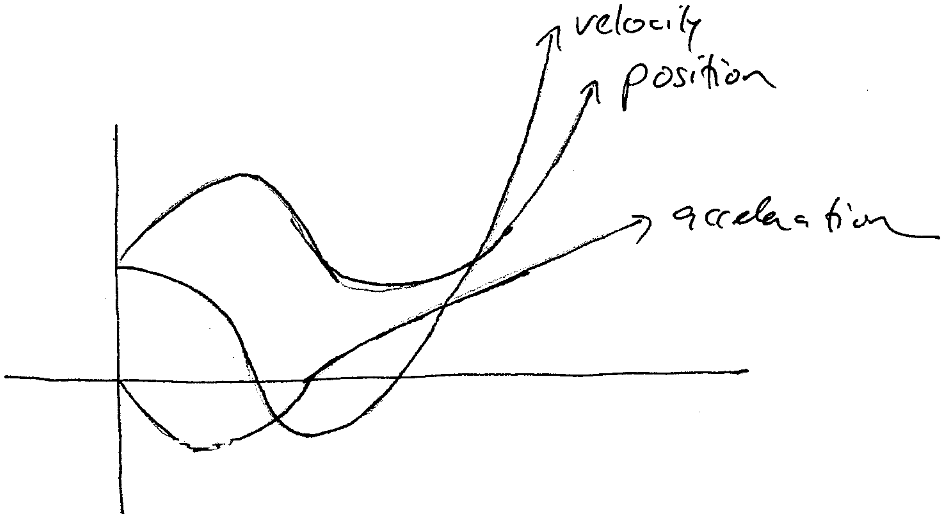
(a) $g(\underline{3}) = \underline{7}$

(b) $g'(\underline{3}) = \underline{2}$

$$g'(3) = \text{slope of tangent line} = \frac{7.04-7}{3.02-3} = \frac{.04}{.02} = 2$$



(3) (10 pts) The figure below shows the graph of three functions. One is the position function of a car, one is the velocity of the car, and one is its acceleration. Label each curve.



(4) (10 pts) USE THE DEFINITION OF DERIVATIVE to compute the derivative of the function $f(x) = 1 - x^2$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{1 - (x+h)^2 - (1 - x^2)}{h}$$

$$= \lim_{h \to 0} \frac{1 - x^2 - 2xh - h^2 - 1 + x^2}{h} = \lim_{h \to 0} \frac{-2xh - h^2}{h}$$

$$= \lim_{h \to 0} -2x - h = -2x$$

(5) (10 pts) Let w be the weight, in pounds, of a refrigerated turkey. Let $f(w)$ be the time, in minutes, required to cook the turkey completely in a 400° oven. Explain, in words, the meaning of the derivative $\frac{df}{dw}$. Do you expect it to be positive or negative? Why?

Roughly, $\frac{df}{dw}$ tells us how many minutes longer we'd have to cook a turkey that's 1 lb. heavier than the one we have now. It should be positive, since it takes longer to cook a heavier turkey.

(6) (15 pts) Compute the derivatives of the following functions. YOU DON'T NEED TO SIMPLIFY YOUR ANSWERS, EXCEPT FOR PART (E).

(a) $\left(\cos x^2 - \cos^2 x\right)' = -\sin x^2 \cdot 2x - (2\cos x \cdot (-\sin x))$ Chain rule

$= -2x \sin x^2 + 2\cos x \sin x$

(b) $\left(x^{2.1} + 3(2.1)^x\right)' = 2.1 x^{1.1} + 3(\ln 2.1)(2.1)^x$ power rule exponential

(c) $\left(\dfrac{4x^3 - 2}{\sin x}\right)'$ $\dfrac{u}{v}$ $= \dfrac{\sin x\,(12x^2) - (4x^3 - 2)\cos x}{\sin^2 x}$ quotient rule

(d) $\left(e^{-x}\sin x \cos x\right)' = e^{-x}(\sin x \cos x)' - e^{-x}\sin x \cos x$ product rule twice

$= e^{-x}(\sin x(-\sin x) + \cos x \cos x) - e^{-x}\sin x \cos x$

$= e^{-x}(\cos^2 x - \sin^2 x - \sin x \cos x)$

(e) $\ln\sqrt{3x}$ $(x > 0)$ (Your answer shouldn't have any $\sqrt{\ }$'s in it.)

$\left(\ln(3x)^{\frac{1}{2}}\right)' = \dfrac{1}{(3x)^{\frac{1}{2}}} \cdot \left((3x)^{\frac{1}{2}}\right)' = \dfrac{1}{\sqrt{3x}} \cdot \dfrac{1}{2}(3x)^{-\frac{1}{2}} \cdot 3 = \dfrac{1}{\sqrt{3x}} \cdot \dfrac{1}{2} \cdot \dfrac{1}{\sqrt{3x}} \cdot 3$

$= \dfrac{3}{2} \cdot \dfrac{1}{3x} = \dfrac{1}{2x}$ chain rule

(7) (16 pts) A parachutist jumps out of a plane flying at 10,000 feet. Let $f(t)$ be the parachutist's height above ground, in feet, t seconds after she jumps out. Label each expression on the left with the letter, or letters, of the matching physical quantity on the right. If it doesn't match any of the physical quantities, label it "NONE." (Each expression may match more than one physical quantity, or it may match none. Similarly, each physical quantity may match more than one expression, or it may match none.)

(i) $f(2)$ A

(ii) $f''(2)$ F

(iii) $f(2) - f(0)$ NONE

(iv) $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$ C

(v) $\frac{f(2) - f(0)}{2}$ G

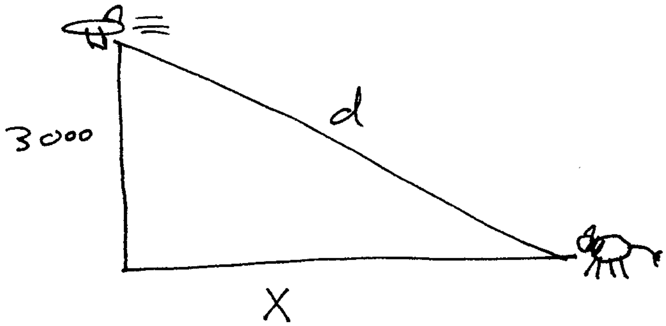
(vi) $\lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$ NONE

(vii) $10,000 - f(2)$ NONE

(viii) $f'(2)$ C

(a) Height after 2 seconds
(b) Time when parachutist hits the ground
(c) Instantaneous velocity after 2 seconds
(d) Average height over first 2 seconds
(e) Instantaneous acceleration after t seconds
(f) Instantaneous acceleration after 2 seconds
(g) Average velocity over first 2 seconds
(h) Rate of change of acceleration after 2 seconds

(8) (17 pts) An airplane flying horizontally at an altitude of 3000 meters and at a constant speed of 200 meters per second passes directly over a ferocious lion taking a nap in a field. How fast is the distance from the plane to the lion increasing ~~30~~ 20 seconds later?



$x' = 200$

$x = 200 \cdot 20 = 4000$

$$x^2 + (3000)^2 = d^2$$

$$\frac{d}{dt}\left[x^2 + (3000)^2 = d^2\right]$$

$$2x\frac{dx}{dt} + 0 = 2dd'$$

$$\frac{xx'}{d} = d'$$

when $x = 4000$, $d = \sqrt{4000^2 + 3000^2} = 5000$

So at that instant $d' = \frac{4000 \cdot 200}{5000}$

$$= \frac{4}{5} \cdot 200 = 160 \text{ m/s}$$