7. Find all of the critical points of the function $f(x) = x^{\frac{2}{3}} - \frac{1}{3}x^2$. (15 points)

$$f'(x) = \frac{2}{3}x^{-\frac{1}{3}} - \frac{2}{3}x.$$

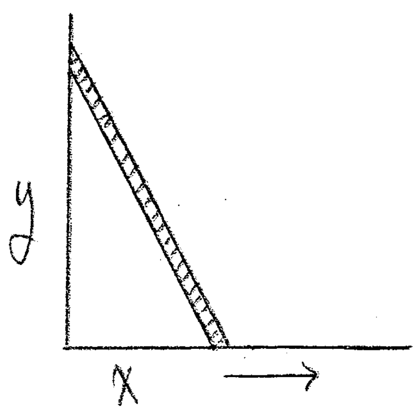
Note that f' is undefined when $x = 0$.

$f'(x) = 0$ when $x^{-\frac{1}{3}} = x$. So $x^{\frac{4}{3}} = 1$, or $x^4 = 1$.

So $f'(x) = 0$ when $x = 1$ and when $x = -1$

There are three critical points: $(0,0)$, $(1, \frac{2}{3})$ and $(-1, \frac{2}{3})$.

8. Consider a ladder 26 ft long which leans against a vertical wall. At a particular instant, the foot of the ladder is 10 ft from the base of the wall and is being drawn away from the wall at a rate of 4 ft/s. How fast is the top of the ladder moving down the wall at this instant?
Useful fact: $(10)^2 + (24)^2 = (26)^2$. (15 points)



Let $y(t)$ = distance from the top of the ladder to the ground at time t.

Let $x(t)$ = distance from the wall to the base of the ladder at time t.

$x^2 + y^2 = (26)^2$, so $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$.

Hence, $\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$. When $x = 10$, $y = 24$ and

$\frac{dx}{dt} = 4$ ft/s so $\frac{dy}{dt} = -\frac{10}{24}(4) = -\frac{40}{24} = -\frac{5}{3}$ ft/s

The top of the ladder moves down at $5/3$ ft/s

9. Suppose that at time t (where t is measured in years), the population of squirrels on the Swarthmore College campus is increasing at a rate of $6(1.03)^t$ squirrels per year. Which of the following expressions represents the total increase in the squirrel population between the fourth and tenth years? (10 points)

(a) $6((1.03)^{10} - (1.03)^4)$

(b) $6(\ln 1.03)((1.03)^{10} - (1.03)^4)$

(c) $\int_4^{10} 6(1.03)^t dt$

(d) $\int 6(1.03)^t dt$

(e) $\frac{1}{6}\int_4^{10} 6(1.03)^t dt$

(f) none of the above

10. Flea F. Hutton jumps straight up off the ground. Her initial upward speed is 96 ft/sec. (20 points)

(a) Find formulas for her upward velocity $v(t)$ and height above the ground $s(t)$ at time t. (Recall that the acceleration due to gravity is -32 ft/sec^2.)

$v(t) = -32t + 96$

$s(t) = -16t^2 + 96t$

(b) How high does she get, and when does she reach the highest point in her leap?

$v(t) = 0$ when $t = 3$. She reaches the highest point when $t = 3$ s. Her height when $t = 3$ is

$s(3) = -16(3^2) + 96(3) = -16(9) + 16(6)(3) = 16(9) = 144$

(c) How long is she in the air?

She is in the air 6 seconds – three seconds going up and 3 seconds coming down

11. Let $v(t)$ denote the velocity of a particle at time t. Correctly label each of the mathematical expressions below with one of the following phrases:

"displacement from time 4 to time 10"
"average velocity from 4 to 10"
"acceleration at time 4"
"v(x)"
"v(10) - v(4)"
"none of the above"

Please write out the correct phrase next to each expression. Do not use arrows and do not number the phrases and refer to them by number. (10 points)

Mathematical expressions:

$\int_4^x v'(t)dt$ none of the above

$v'(4)$ = acceleration at time 4.

$\frac{v(4)+v(10)}{2}$ none of the above

$\int_4^{10} v(t)dt$ = displacement from time 4 to time 10

$\int_4^{10} v'(t)dt$ = $v(10) - v(4)$

$\frac{1}{6}\int_4^{10} v(t)dt$ = average velocity from 4 to 10

$\frac{d}{dt}\int_4^{10} v(t)dt$ none of the above

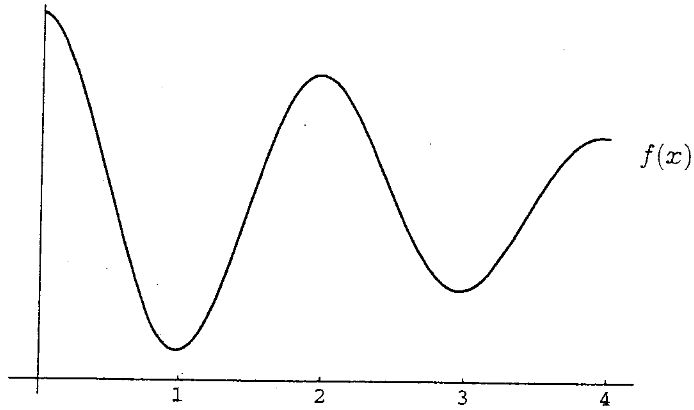
$\frac{d}{dt}\int_4^{x} v(t)dt$ = $v(x)$

$\lim_{h\to 0} \frac{v(4+h)-v(4)}{h}$ = $v'(4)$ = acceleration at time 4

$\lim_{h\to 0} \frac{v(4+h)+v(4)}{h}$ none of the above

$\lim_{h\to 0} \frac{v'(4+h)-v'(4)}{h}$ = $v''(4)$. none of the above

12. Consider the graph of the function $f(x)$ shown below. (15 points)

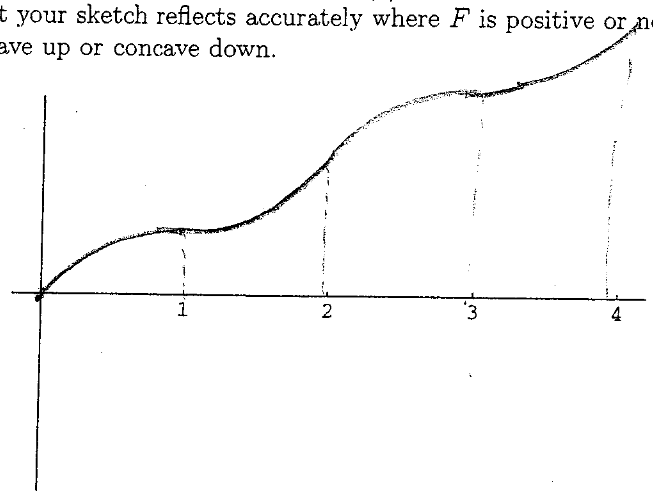


(a) Sketch the graph of a function F such that $F' = f$ and $F(0) = 0$. Where there is enough information to do so, make sure that your sketch reflects accurately where F is positive or negative, increasing or decreasing, and concave up or concave down.

F is increasing for all x

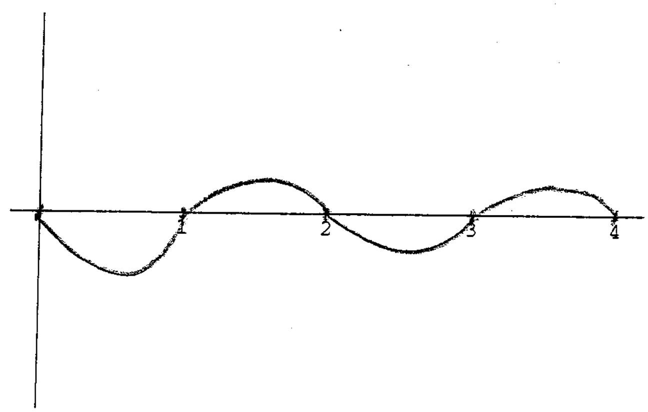
F is concave down on [0,1] and [2,3].

F is concave up on [1,2] and [3,4]



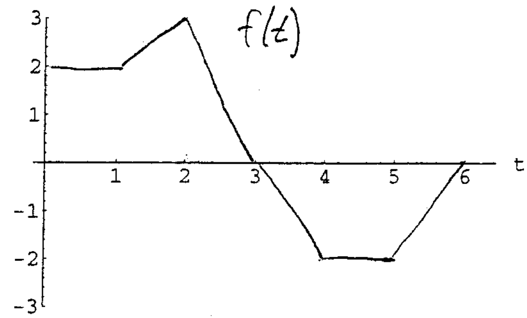
(b) Sketch the graph of f', paying attention, where possible, to the same sort of details as in part (a). In addition, can you determine whether the integral of f' from $x = 0$ to $x = 4$ is positive, negative, or zero? If so, which is it? Or is more information needed before you can tell for sure? Explain clearly.

$f' = 0$ at 0, 1, 2, 3 and 4



$\int_0^4 f'\,dx = f(4) - f(0)$. The graph of f shows $f(4) < f(0)$ so $f(4) - f(0) < 0$. So $\int_0^4 f'\,dx < 0$.

13. Let $g(x) = \int_0^x f(t)\,dt$, where $f(t)$ is the function graphed below. (15 points)



(a) Evaluate each of the following:

(i) $g(0)$ $= \int_0^0 f(t)\,dt = 0$

(ii) $g(1)$ = area of 1×2 rectangle $= 2$

(iii) $g(2)$ $= 4.5 =$ area of 2×2 square + area of triangle with $b = h = 1$.

(iv) $g(3)$ $= g(2) +$ (area of triangle of height 3 and base 1) $= 4.5 + \frac{3}{2} = 6$

(v) $g(6)$ $= g(3) -$ (area of region below the axis) $= 6 - (1+2+1) = 2$

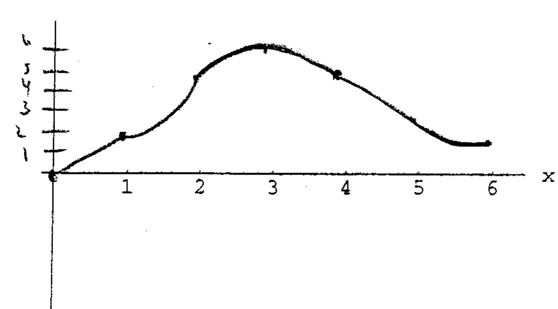
(b) On what interval(s) is g increasing?

g is increasing whenever $f > 0$. So g increases on $[0,3]$

(c) At what x value does g have a local maximum?

g has a local maximum at $t = 3$.

(d) Sketch a graph of g. Be sure to label the scale on the y-axis.



g is a straight line on $[0,1]$ and $[4,5]$

It's concave up on $[1,2]$ and $[5,6]$ and concave down on $[2,4]$