1. Find the derivatives of the following functions. (20 points)

(a) $y = 3^x$ $\quad 3 = e^{\ln 3}$, so $y = (e^{\ln 3})^x = e^{(\ln 3)x}$

Then $y' = (\ln 3)e^{(\ln 3)x}$

$= (\ln 3)3^x$

(b) $y = 3x^5 \sin(4x)$

$$y' = (3x^5)(\sin(4x))' + (3x^5)'(\sin(4x))$$
$$= 3x^5 \cdot 4 \cdot \cos 4x + 15x^4 \sin(4x)$$

(c) $y = \dfrac{3x+4}{x^3+x+2}$ $\quad y' = \dfrac{(x^3+x+2)\cdot 3 - (3x+4)(3x^2+1)}{(x^3+x+2)^2}$

(d) $y = \left(\ln(3x^2+4)\right)^8$ $\quad y' = 8(\ln(3x^2+4))^7 \cdot (\ln(3x^2+4))'$

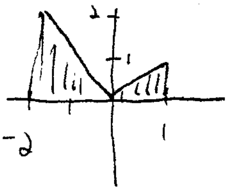
$$= 8(\ln(3x^2+4))^7 \cdot \frac{6x}{3x^2+4}$$

2. Evaluate the following integrals. (20 points)

(a) $\int \left(x^{\frac{3}{2}} + \sin x\right) dx$

$$\frac{2}{5} x^{\frac{5}{2}} - \cos x + C$$

(b) $\int_{-2}^{1} |x|\, dx$



$\int_{-2}^{1} |x|\, dx$ = area under curve

$= \frac{1}{2}(2)(2) + \frac{1}{2}(1)(1)$

$= 2 + \frac{1}{2}$

(c) $\int \frac{1}{x}\, dx$

$$\ln |x| + C$$

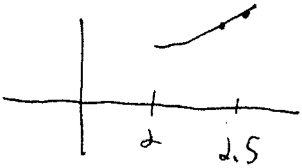
(d) $\int_{\ln 3}^{\ln 6} 8e^t\, dt$ $= \left. 8e^t \right|_{\ln 3}^{\ln 6} = 8e^{\ln 6} - 8e^{\ln 3}$

$= 8 \cdot 6 - 8 \cdot 3 = 24$

3. Below is a table of some values for the function $f(x) = x^x$. (20 points)

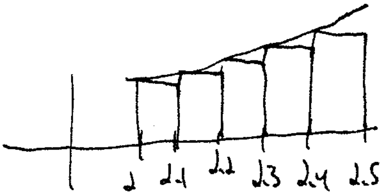
x	2	2.1	2.2	2.3	2.4	2.5
x^x	4	4.7	5.7	6.8	8.2	9.9

(a) Use the values in the table to compute an estimate for $f'(2.5)$.



$$f'(2.5) \approx \frac{f(2.5)-f(2.4)}{0.1} = \frac{9.9-8.2}{0.1} = \frac{1.7}{0.1} = 17$$

(b) Use the values in the table to compute a left hand sum to estimate $\int_2^{2.5} x^x\,dx$.

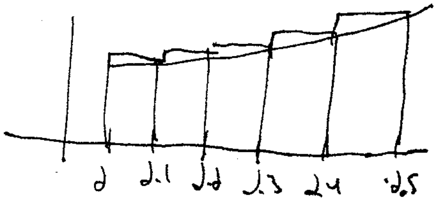


Area of rectangles is

$.1\times 4 + .1\times 4.7 + .1\times 5.7 + .1\times 6.8 + .1\times 8.2$

$= 2.94$

(c) Use the values in the table to compute a right hand sum to estimate $\int_2^{2.5} x^x\,dx$.



Area is

$.1\times 4.7 + .1\times 5.7 + .1\times 6.8 + .1\times 8.2 + .1\times 9.9$

$= 3.53$

(d) How small should Δx be to guarantee that the left and right hand sums for $\int_2^{2.5} x^x\,dx$ differ by at most 0.01?

$RHS - LHS = (f(2.5) - f(2))\Delta x$

$= 5.9\Delta x$

To make $5.9\Delta x \le 0.01$, make $\Delta x \le \frac{0.01}{5.9} = \frac{1}{590}$

4. Suppose that at a price of \$$p$ per pound, q pounds of delicious cheese are sold. Let $q = f(p)$. If $f(10) = 300,000$ and $f'(10) = -40,000$, roughly how many pounds of delicious cheese would you expect to be sold if the price p is \$9.50? (10 points)

$$f(p) \approx f(10) + f'(10)(p-10)$$

So $f(9.5) \approx 300,000 - 40,000(-0.5)$

$$= 300,000 + 20,000$$

$= 320,000$ lbs. of delicious cheese

5. Find an equation for the tangent line to the curve $x^2 + 3y^2 = 4$ at the point $(1, 1)$. (10 points)

$y-1 = m(x-1)$, where m is the slope $\frac{dy}{dx}$ at $(1,1)$.

Use implicit differentiation: $x^2+3y^2 = 4$

so $\frac{d}{dx}(x^2+3y^2) = \frac{d}{dx}(4)$

$$2x + 6y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x}{3y}$$

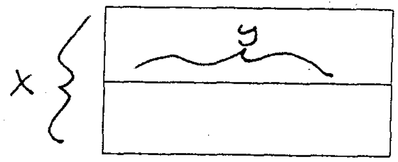
At $(1,1)$, $\frac{dy}{dx} = \frac{-1}{3}$, so the line is

$$y-1 = \frac{-1}{3}(x-1)$$

6. Do **ONE** of the following two problems (either A or B). If you try both, be sure to say which one you want us to grade.

(20 points)

(A) A rectangular plot of land of area 600 ft^2 is to be surrounded by a stone wall and then divided into two equal parts by a fence parallel to one side. If it costs \$10 per foot to build the stone wall and \$2 per foot to build the fence, what should the dimensions of the plot be in order for the total cost to be as small as possible?



(B) The manager of a 100-unit apartment building knows that all units will be occupied if the rent is \$400 per month, and that one additional unit will remain vacant for each \$5 increase in rent. What rent should the manager charge to maximize revenue?

A) minimize cost $= 10(\text{outside length}) + 2(\text{inside length}) = 10(2x+2y) + 2y$

$= 20x + 22y$

Since area $= 600 = xy$, we have $y = \frac{600}{x}$, and cost is $C(x) = 20x + \frac{22\cdot 600}{x}$. To minimize, check "endpoints" 0 & ∞ and critical pts. At 0 & ∞, the cost is infinite, so the minimum must occur at a critical pt. $C'(x) = 20 - \frac{22\cdot 600}{x^2}$. $C'(x)$ doesn't exist at $x=0$ (which we already know isn't a min). Set $C'(x)=0$: $0 = 20 - \frac{22\cdot 600}{x^2}$, so $\frac{11\cdot 60}{x^2} = 1$, so $11\cdot 60 = x^2$, so $x = \sqrt{11\cdot 60}$ (the $-$ square root doesn't make sense), and $y = \frac{600}{\sqrt{11\cdot 60}}$

B) Max. revenue = (# apts. occupied)(rent) $= n\cdot r$. Let x = # of \$5 rent increases. Then $r = 400+5x$, and $n = 100-x$. So revenue $= R(x) = (400+5x)(100-x)$ $= -5x^2 + 100x + 40{,}000$. Endpts are $x=0$ ($R(0) = 40{,}000$) and $x=100$ (no apts. rented: $R(100)=0$). Find critical pts: set $R'(x)=0$.
$R'(x) = -10x + 100 = 0$, so $x = 10$. $R(10) = (450)(90) = 40{,}500 \leftarrow$ max.

So charge rent $400 + 5(10) = \$450$.