Previous Final Exam Questions

1. Compute the following.

a) $\frac{d}{dx}(3x^2-7)$ b) $\int(3x^2-7)\,dx$ c) $\frac{d}{dx}(\sin(2x))$ d) $\int_0^{\frac{\pi}{4}}\sin(2x)\,dx$ e) $\frac{d}{dx}\left(\frac{x+1}{(x-1)^2}\right)$

2. For each of the following, find $\frac{dy}{dx}$.

a) $y=\sin(\sin(x))$ b) $y=\arctan(e^x)$ c) $y=x\tan x$ d) $y=\ln(\frac{1}{x})$ e) $x^2+3xy+y^2=1$

3. In this problem, you are given the following data about a function f.

x	6.5	7	7.5	8	8.5	9
$f(x)$	10.3	11.3	12.1	12.3	11.2	9.3

a) Does this data suggest that $f''(7)$ is positive or that $f''(7)$ is negative? Explain.

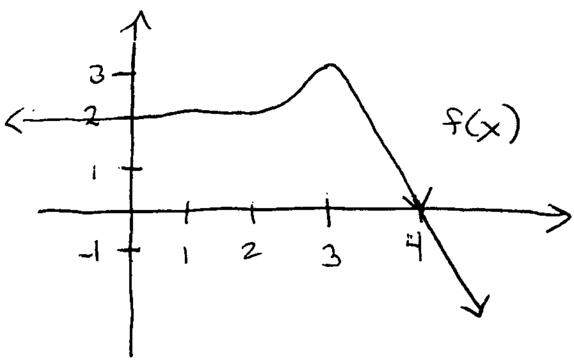
b) Estimate $f'(7)$.

c) Use the estimation you gave in part b) to give an equation of a tangent line approximation to $f(x)$ which is good for x near 7.

d) Estimate $f(7.1)$ using part (c).

e) Indicate whether the estimation you just gave is likely to be too big or too small. Explain.

4. The function $f(x)$ is defined by the graph given below. $F(x)$ is a function which has derivative $f(x)$ and which satisfies $F(0)=0$.



a) Use the graph to estimate $F(1.5)$ and $F(3)$.

b) Sketch the graph of $F(x)$.

c) Sketch a graph of a *different* function $G(x)$ which also has derivative $f(x)$.

5. In this problem, you are given the following data about the sign of the function $f(x)$ and of its first and second derivatives.

x	$x<0$	$x=0$	$0<x<1$	$x=1$	$1<x<2$	$x=2$	$2<x<3$	$x=3$	$3<x<4$	$x=4$	$x>4$
$f(x)$	−	0	+	+	+	+	+	+	+	0	−
$f'(x)$	+	+	+	0	−	−	−	0	−	−	−
$f''(x)$	−	−	−	−	−	0	+	0	−	−	−

a) Find all the critical points of $f(x)$ and classify each one as a maximum or minimum or neither,

b) Find all inflection points of $f(x)$.

c) Use the information above to graph $f(x)$.

6. A city bus stop shelter consisting of 2 square sides, a rectangular back, and a flat roof is to have a volume of 2000 ft^3. Find the dimensions which will use the least amount of material. (Use techniques of calculus to show your answer is indeed a minimum.)

7. Let f be increasing everywhere. Which Riemann sum (left-hand or right-hand) gives an underestimate for the value of $\int_a^b f(x)\,dx$? Explain your answer using good English and an illustrative picture.

8. Farmer Jane is deciding how much fertilizer to use on her crops. Suppose that $y = f(x)$ is the function that determines Jane's crop yield, measured in bushels, as a function of the amount of fertilizer, measured in pounds, that Jane uses on her land.

   a) Jane's almanac says, "Applying a small amount of fertilizer will produce dramatic improvements in crop yield, as compared with using no fertilizer at all. However, as more fertilizer is used, the rate of improvement in crop yield becomes smaller." Interpret this statement in terms of signs of the first and second derivatives of f.

   b) Interpret the statement $f'(100) = 2.5$ in the language of fertilizer and crop yield.

9. Label each of the following as "TRUE" or "FALSE":

   a) $$\int (x^2+1)^9(2x)dx = \frac{(x^2+1)^{10}}{10} + C$$

   b) $$\int 2^x dx = \frac{2^{(x+1)}}{(x+1)} + C$$

   c) $$\int \sin(x)\cos(x)dx = \frac{\sin^2(x)}{2} + C$$

10. Suppose that the function $f(x)$ is increasing on the interval $[0, 8]$, with $f(0) = 10$ and $f(8) = 40$. We wish to evaluate

$$\int_0^8 f(x)\,dx$$

but we cannot find an antiderivative of $f(x)$. Explain how you would find an estimate for this integral with an error of at most 10^{-4}.

11. Let

$$g(x) = \int_1^{x^2-4x} te^t\,dt.$$

Find the x-coordinates of the local maxima and minima of $g(x)$. Be sure to say which is which!

12. You are standing on top of a mound of dirt, and you toss the ball up into the air. After 1/2 second, the ball is moving upward at 8 ft/s. The ball hits the ground with a speed of 40 ft/s. What is the height of the mound?

13. Evaluate the following limit: $\lim_{x\to 0} \frac{3^{\sin x} - 1}{\sin x}$.

14. True or False? $\int_{-2}^{1} \frac{1}{\sqrt{x^4+9}}\,dx = -\frac{3}{8}$. (Do not guess. Give reasoning.)

15. You are videotaping a race from a stand 132 feet from the track, following a car that is moving at a constant velocity along a straight track. When the car is directly in front of you (i.e. when $\theta = 0$) the angle θ is changing at a rate of

$$\left.\frac{d\theta}{dt}\right|_{\theta=0} = \frac{24}{132}$$

How fast is the car going? How fast will the camera angle θ be changing a half second later?

16. The population, in millions, of a country at time t is given by the function

$$P(t) = 100e^{.02t}.$$

Find the average size of the population over the interval $[0, 10]$.