Math 18, Solns to selected HW problems, §8.4, due 12/11/01

(2) S = unit sphere, D = unit ball $\iint_S \vec{F}\cdot d\vec{S} = \iiint_R \nabla\cdot\vec{F}\,dV = \iiint_D (3x^2+3y^2+3z^2)\,dV$
$= \frac{12\pi}{5}$ (use spherical coords)

(8) D = unit ball $\iint_S \vec{F}\cdot d\vec{S} = \iiint_D \nabla\cdot\vec{F}\,dV = \iiint_D (3y^2+3x^2+3z^2)\,dV$
$= 12\pi/5$ again

(10) $\iint_{\partial S} \vec{F}\cdot\vec{n}\,dA = \iiint_S \nabla\cdot\vec{F}\,dV$ (S cylinder)
$= \iiint_S (x^2+y^2)^2\,dV = \int_0^1\int_0^{2\pi}\int_0^1 (r^2)^2\,r\,dr\,d\theta\,dz$

$= \pi/3$

(18) $\iiint_\Omega \nabla\cdot\vec{F}\,dV = \iiint_S \vec{F}\cdot d\vec{S}$ (Gauss' Thm)

$= \iiint_S \vec{F}\cdot\vec{n}\,dA$

Since $\vec{F}$ is tangent to S, $\vec{F}\cdot\vec{n}=0$, so $\iiint_S \vec{F}\cdot\vec{n}\,dA = 0$