Math 18, Solns. to selected HW problems, §8.2–3, due 12/5/01

8.2.10, 8.3.2

**8.2.10** By Stokes' Theorem, $\iint_S (\nabla \times F)\cdot d\vec{S} = \int_C \vec{F}\cdot ds$, where $C$ is the boundary curve of $S$. Since $S$ has no boundary, $C$ is empty, so $\int_C \vec{F}\cdot ds = 0$.

 ← $S$ is a squashed sphere – no boundary curve/edge

**8.3.2** a) $\int_C \vec{F}\cdot ds = \int_0^1 \left(x(2x^2), (2x^2)^2\right)\cdot(1, 4x)\,dx$

(since $c(x) = (x, 2x^2)$, $0 \le x \le 1$).

$$= \int_0^1 (2x^3 + 16x^5)\,dx = \left(\frac{x^4}{2} + \frac{8}{3}x^6\right)\Big|_0^1 = \frac{1}{2} + \frac{8}{3} = \frac{3}{6} + \frac{16}{6} = \frac{19}{6}$$

---

b) Since $\nabla \times \vec{F} = \det\begin{pmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & y^2 & 0 \end{pmatrix} = i(0-0) - j(0-0) + k(0-x) = (0, 0, -x)$

is not $\vec{0}$, we expect that the integral <u>will</u> depend on the path. And in fact if we take $C_1$ to be the path $C_1(x) = (x, 2x)$, then we get $\int_{C_1} \vec{F}\cdot ds = \frac{10}{3}$,

which is different from the $\frac{19}{6}$ we got on the first path, $C$.