Math 18, Solns. to selected HW problems, §7.6 & 8.1, due 11/28

7.6.9, 8.1.2, 8.1.13

7.6.9 Parametrize the sphere the usual way: $x=\sin\phi\cos\theta$, $y=\sin\phi\sin\theta$, $z=\cos\phi$, $0\le\phi\le\pi$, $0\le\theta\le 2\pi$. Then

$$\iint_S \vec{F}\cdot d\vec{S} = \int_0^{2\pi}\int_0^{\pi} \left(3\sin\phi\cos\theta(\sin\phi\sin\theta)^2,\ 3(\sin\phi\cos\theta)^2(\sin\phi\sin\theta),\ \cos^3\phi\right) \cdot (T_\phi \times T_\theta)\, d\phi\, d\theta,$$

where $T_\phi = (\cos\phi\cos\theta, \cos\phi\sin\theta, -\sin\phi)$ & $T_\theta = (-\sin\phi\sin\theta, \sin\phi\cos\theta, 0)$.

This integral evaluates to $12\pi/5$ (Mathematica is your friend).

8.1.2 Area $= \frac{1}{2}\int_C -y\,dx + x\,dy$, where C is the unit circle oriented CCW, ie, $x=\cos t$, $y=\sin t$, $0\le t\le 2\pi$

So Area $= \frac{1}{2}\int_0^{2\pi}\left[-\sin t(-\sin t) + \cos t(\cos t)\right] dt$

$= \frac{1}{2}\int_0^{2\pi}(\sin^2 t + \cos^2 t)\,dt = \frac{1}{2}\int_0^{2\pi} 1\,dt = \pi$

8.1.13 By Green's Theorem, this integral equals $\iint_D (-2y + 4x^3)\,dy\,dx$, where D is the unit square. This equals $\int_0^1\int_0^1 (-2y+4x^3)\,dy\,dx$

$= \int_0^1 \left[-y^2 + 4x^3y\right]_0^1 dx$

$= \int_0^1 \left[-1+4x^3\right]dx = -x+x^4\Big|_0^1 = 0$