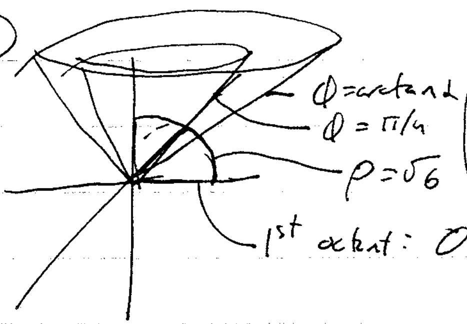
Math 18, Solns. to selected HW problems, §§6.1-3 & 7.1

#6.2.30, 6.3.12, 7.1.2a)

6.2.30



so: $\int_0^{\sqrt{6}} \int_{\pi/4}^{\arctan 2} \int_0^{\frac{\pi}{2}} \frac{1}{\rho} \cdot \rho^2 \sin\phi \, d\theta \, d\phi \, d\rho$

$= \int_0^{\sqrt{6}} \int_{\pi/4}^{\arctan 2} \frac{\pi}{2} \rho \sin\phi \, d\phi \, d\rho$

$= \int_0^{\sqrt{6}} \frac{\pi}{2} \rho \left(\cancel{\cos \pi/4} - \cos(\arctan 2) + \cos \pi/4 \right) d\rho$

$= \int_0^{\sqrt{6}} \frac{\pi}{2} \rho \left(-\frac{1}{\sqrt{5}} + \frac{1}{\sqrt{2}} \right) d\rho = \boxed{\frac{\pi}{2} \left(\frac{-1}{\sqrt{5}} + \frac{1}{\sqrt{2}} \right) \cdot 3}$

6.3.12 Average over B is $\frac{\iiint_B e^{-z} \, dV}{\text{volume of } B} = \frac{\iiint_B e^{-z} \, dV}{\frac{4}{3}\pi}$

In cylindrical coordinates, $\iiint_B e^{-z} \, dV = \int_0^{2\pi} \int_0^1 \int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} e^{-z} r \, dz \, dr \, d\theta$

$= \int_0^{2\pi} \int_0^1 r \left(-e^{-\sqrt{1-r^2}} + e^{\sqrt{1-r^2}} \right) dr \, d\theta = \cancel{\int_0^{2\pi} \left(e^{-\sqrt{1-r^2}} + e^{\sqrt{1-r^2}} \right) \Big|_0^1 d\theta}$

(Mathematica)

$\cancel{\int_0^{2\pi} \ldots \, d\theta} = \int_0^{2\pi} \frac{2}{e} \, d\theta = \frac{4\pi}{e}.$

So the av. is $\frac{\frac{4\pi}{e}}{\frac{4\pi}{3}} = \frac{3}{e}.$

7.1.2a) $\int_0^{2\pi} (\sin t + \cos t + t) \|c'(t)\| \, dt$ $\quad c'(t) = (\cos t, -\sin t, 1)$

$\|c'(t)\| = \sqrt{\cos^2 t + \sin^2 t + 1^2} = \sqrt{2}$

$= \int_0^{2\pi} \sqrt{2} (\sin t + \cos t + t) \, dt$

$= \sqrt{2} \left(-\cos t + \sin t + \frac{t^2}{2} \right) \Big|_0^{2\pi} = \sqrt{2} \cdot \frac{4\pi^2}{2} = 2\sqrt{2}\,\pi^2$