Math 18 Solutions to selected HW problems, §§3.6, 4.1, 4.2, due 10/12

3.6 # 10, 4.1 # 18, 4.2 # 12

3.6.10 Maximize $Q(x,y) = xy$, subject to the constraint $C(x,y) = 2x + 3y = 10$.

Check: $\nabla C = \vec{0}$ $\quad \nabla C = (2,3) \neq \vec{0}$

& $\nabla Q = \lambda \nabla C$

$(y,x) = \lambda(2,3)$ $\quad y = 2\lambda \quad x = 3\lambda$, so $3y = 2x = 6\lambda$

$y = \frac{2}{3}x$

$2x + 3y = 10, \quad 2x + 3\cdot\frac{2}{3}x = 10, \quad 4x = 10, \quad x = \frac{5}{2}, \quad y = \frac{5}{3}$

So the max. quantity is $Q\left(\frac{5}{2}, \frac{5}{3}\right) = \frac{5}{2}\cdot\frac{5}{3} = \frac{25}{6}$

4.1.18 Let $c(t) = (x(t), y(t), z(t))$.

Since $c''(t) = (x''(t), y''(t), z''(t)) = (0,0,0)$, it must be true that $x'(t)$, $y'(t)$, and $z'(t)$ are all constants (since their derivatives are 0). So let $x'(t) = C_1$, $y'(t) = C_2$, and $z'(t) = C_3$. Then, by integrating, we get that $x(t) = C_1 t + d_1$, $y(t) = C_2 t + d_2$, and $z(t) = C_3 t + d_3$. This is the eqn. of a line, unless $C_1 = C_2 = C_3 = 0$, in which case it's a point.

4.2.12 a) $T(t)\cdot T(t) = \|T(t)\|^2 = 1$. Differentiating the L & R sides:

$T'(t)\cdot T(t) + T(t)\cdot T'(t) = \frac{d}{dt}1 = 0$

So $T'(t)\cdot T(t) = 0$

b) $T(t) = \dfrac{c'(t)}{(c'(t)\cdot c'(t))^{\frac{1}{2}}}$, so

$$T'(t) = \frac{(c'(t)\cdot c'(t))^{\frac{1}{2}} c''(t) - c'(t)\cdot\frac{1}{2}(c'(t)\cdot c'(t))^{-\frac{1}{2}}\cdot(c''(t)\cdot c'(t) + c'(t)\cdot c''(t))}{\left((c'(t)\cdot c'(t))^{\frac{1}{2}}\right)^2}$$

$$= \frac{\|c'(t)\|\, c''(t) - c'(t)\cdot\frac{1}{\|c'(t)\|}\cdot(c'(t)\cdot c''(t))}{\|c'(t)\|^2}$$