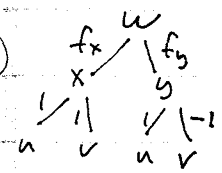
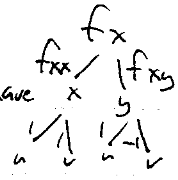
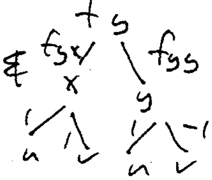
Math 18 Solns to selected HW problems, §§ 3.1–3.4, due 10/3/01

3.1 #12, 3.3 #2, 3.4 #20

3.1.12 

$\frac{\partial w}{\partial v} = f_x \cdot 1 + f_y(-1) = f_x - f_y$. We have  & ,

so $\frac{\partial^2 w}{\partial u \partial v} = \frac{\partial}{\partial u}\left(\frac{\partial w}{\partial v}\right) = \frac{\partial}{\partial u}(f_x - f_y) = f_{xx}\cdot 1 + f_{xy}\cdot 1 - (f_{yx}\cdot 1 + f_{yy}\cdot 1)$

$= f_{xx} + f_{xy} - f_{yx} + f_{yy} = f_{xx} + f_{yy}$ (assuming f is of class C^2).

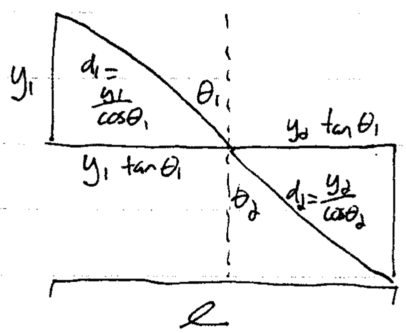
3.3.2 $f(x,y) = x^2 + y^2 - xy$ $\quad f_x = 2x - y \quad f_y = 2y - x$

$Df(x,y) = [2x-y \;\; 2y-x] = [0 \;\; 0]$, so $2x = y$ & $x = 2y$. Thus $x = y = 0$ is the only critical pt.

$A = f_{xx} = 2 \quad B = f_{xy} = -1 \quad C = f_{yy} = 2 \quad \Delta = AC - B^2 = 4 - 1 = 3$

So $(0,0)$ is a local minimum.

3.4.20 Let l = horizontal distance b/w A & B, y_1 the vertical distance from A to the boundary, and y_2 the vertical distance from B to the bdy. We have



Our constraint is $g(\theta_1,\theta_2) = y_1 \tan\theta_1 + y_2 \tan\theta_2 = l$.

We want to minimize the time, ie, $\frac{d_1}{v_1} + \frac{d_2}{v_2} = \frac{y_1}{v_1\cos\theta_1} + \frac{y_2}{v_2\cos\theta_2} = f(\theta_1,\theta_2)$.

$\nabla g(x,y) = (y_1 \sec^2\theta_1,\; y_2 \sec^2\theta_2)$, which is never $(0,0)$.

So try to solve $\nabla f(x,y) = \lambda \nabla g(x,y)$

$$\left(\frac{y_1}{v_1}\tan\theta_1 \sec\theta_1,\; \frac{y_2}{v_2}\tan\theta_2\sec\theta_2\right) = \lambda\,(y_1\sec^2\theta_1,\; y_2\sec^2\theta_2).$$

1st equation: $\frac{y_1}{v_1}\tan\theta_1\sec\theta_1 = \lambda y_1 \sec^2\theta_1$ reduces to $\frac{\sin\theta_1}{v_1} = \lambda$.

Similarly, the 2nd equation reduces to $\frac{\sin\theta_2}{v_2} = \lambda$. Thus $\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$,

or $\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$.