Math 18, Solns to selected HW problems, §2.4-2.6, due 9/26

2.4: # 14 2.5: # 12 2.6: # 20

2.4.14 $r(t) = (6t, 3t^2, t^3)$

$r'(t) = (6, 6t, 3t^2)$, so $r'(0) = (6,0,0)$

2.5.12 a) $T'(t) = D(T(\sigma(t))) = DT(\sigma(t)) \cdot D\sigma(t)$

$= \begin{bmatrix} dx & dy & dz \end{bmatrix}_{\sigma(t) = \cos t, \sin t, t} \cdot \begin{bmatrix} -\sin t \\ \cos t \\ 1 \end{bmatrix}$

$= \begin{bmatrix} d\cos t & d\sin t & dt \end{bmatrix} \cdot \begin{bmatrix} -\sin t \\ \cos t \\ 1 \end{bmatrix} = d\cos t(-\sin t) + d\sin t(\cos t) + dt = \boxed{dt}$

b) $T\left(\frac{\pi}{2} + 0.01\right) \approx T\left(\frac{\pi}{2}\right) + T'\left(\frac{\pi}{2}\right) \cdot (0.01) = T\left(\cos\frac{\pi}{2}, \sin\frac{\pi}{2}, \frac{\pi}{2}\right) + d\frac{\pi}{2} \cdot (0.01)$

$= T\left(0, 1, \frac{\pi}{2}\right) + 0.01 \cdot \pi = 0^2 + 1^2 + \left(\frac{\pi}{2}\right)^2 + 0.01 \cdot \pi = \boxed{1 + \frac{\pi^2}{4} + 0.01\pi}$

2.6.20 $z = c - ax^2 - by^2$

Altitude z increases most rapidly in direction of gradient $\nabla f(1,1)$.

$\nabla f = (-2ax, -2by)$ $\nabla f(1,1) = (-2a, -2b)$

So direction is direction of $(-2a, -2b) = \frac{(-a,-b)}{\sqrt{a^2+b^2}}$

A marble would roll in the direction of greatest decrease, ie, direction of $-\nabla f(1,1) = (2a, 2b)$, namely $\frac{(a,b)}{\sqrt{a^2+b^2}}$.