Math 18 Practice Final Solutions

(Answers to odd questions in the book.)

1.46 $(2,-1,1)$ points in the direction of the line, so an answer is

$$\frac{(2,-1,1)\times(1,-1,0)}{\|(2,-1,1)\times(1,-1,0)\|} = \frac{(1,1,-1)}{\sqrt{3}} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$$

2.86 Let $F(x,y,z) = x^3-2y^3+z^3$. The surface is the level surface $F(x,y,z)=0$, so $\nabla F(1,1,1)$ is a normal vector. $\nabla F(1,1,1) = (3x^2, -6y^2, 3z^2)_{(1,1,1)}$ $= (3,-6,3)$. So the plane is $3(x-1)-6(y-1)+3(z-1)=0$.

2.30 a) $\|u\| = \sqrt{1+4+4} = 3$, $u\cdot v = (1,-2,2)\cdot(2,1,-3) = 2-2-6 = -6$

$u\times v = (4,7,5)$, $\frac{u}{\|u\|} = \left(\frac{1}{3}, \frac{-2}{3}, \frac{2}{3}\right)$

b) $\nabla f\cdot\frac{u}{\|u\|} = \left(ye^{xy}\sin(xyz)+e^{xy}\cdot yz\cdot\cos(xyz),\ xe^{xy}\sin(xyz)+e^{xy}xz\cdot\cos(xyz),\ e^{xy}\cdot xy\cdot\cos(xyz)\right)_{(0,1,1)}\cdot\left(\frac{1}{3},\frac{-2}{3},\frac{2}{3}\right)$

$= (1,0,0)\cdot\left(\frac{1}{3},\frac{-2}{3},\frac{2}{3}\right) = \frac{1}{3}$

7.4 $\int_c \vec{F}\cdot ds = \int_t \vec{F}(c(t))\cdot c'(t)\,dt$. Since $\vec{F}(c(t))$ is $\perp$ to $c'(t)$, $\vec{F}(c(t))\cdot c'(t)=0$, and the integral is 0.

7.18 Orient S upwards. Then the normal vector $\vec{n} = (x,y,z)$. So

$\iint_S \vec{F}\cdot d\vec{S} = \iint_S \vec{F}\cdot\vec{n}\,dS = \iint_S (x,y,z)\cdot(x,y,z)\,dS = \iint_S (x^2+y^2+z^2)\,dS$

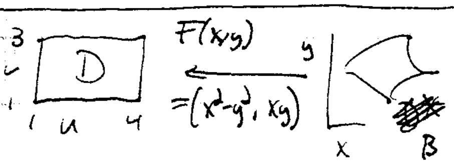
On S, $x^2+y^2+z^2=1$, so this is $\iint_S dS$ = Area of $S = \frac{1}{2}\left(4\pi 1^2\right) = 2\pi$

Extra question $\frac{1}{3}\iint_S \vec{F}\cdot d\vec{S} = \frac{1}{3}\iiint_R (\nabla\cdot\vec{F})\,dV$ by Gauss's Theorem

$$= \frac{1}{3}\iiint_R (1+1+1)\,dV$$

$$= \iiint_R dV = \text{volume of } R$$

6.3



$6 = \text{vol}(D) = \iint_D du\,dv = \iint_B |\det DF|\,dx\,dy = \iint_B \left|\det\begin{pmatrix}2x & -2y\\ y & x\end{pmatrix}\right| dx\,dy$

$= \iint_B 2(x^2+y^2)\,dx\,dy$

So $\iint_R (x^2+y^2)\,dx\,dy = \frac{1}{2}\cdot 6 = 3$