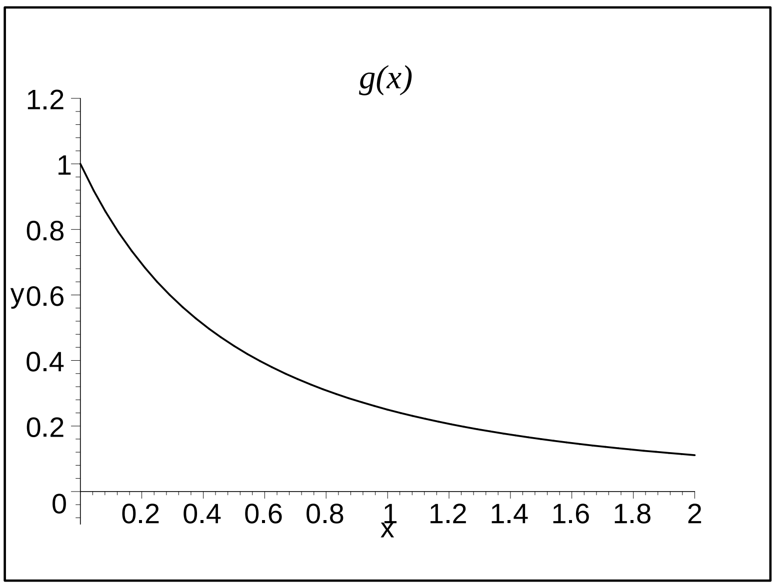
Math 119 Midterm 1 Solutions

1. I dropped a bottle of delicious Vermont maple syrup off a 64-foot tower. How fast was it going when it hit the ground? (Ignore wind resistance, and recall that near the earth's surface the acceleration due to gravity is 32 ft/sec^2.)

First figure out when it hits the ground. $v(t) = \int(-32)\,dt = -32t + C$, **and, since initial velocity is 0,** $C = 0$. **Then the height** $h(t) = \int v(t)\,dt = \int(-32t)\,dt = -16t^2 + C$. **Since the initial height is 64,** $C = 0$, **and** $h(t) = -16t^2 + 64$. **To find when the syrup hits the ground, we solve** $h(t) = 0$, **and take the positive solution,** $t = 2$. **Finally, the velocity after 2 second is** $v(2) = -32(2) = -64$, **so the bottle is moving downwards at 64 feet per second when it hits the ground.**

2. The left, right, trapezoid, and midpoint rules were used to estimate $\int_0^2 g(x)\,dx$, where g is the function whose graph is shown. The estimates were 0.7811, 0.8632, 0.8675, and 0.9540, and the same number of subintervals were used in each case.



(a) Which rule produced which estimate?

(b) Between which two estimates does the true value of $\int_0^2 g(x)\,dx$ lie?

(a) By looking at the slope and concavity of the graph, we see that the left rule overestimates by a lot, the right rule underestimates by a lot, the trapezoid rule overestimates by a little, and the midpoint rule underestimates by a little. So 0.7811 is the right rule, 0.8632 is the midpoint rule, 0.8675 is the trapezoid rule, and 0.9540 is the left rule.

(b) Reasoning as in (a), we see that the real answer is between the midpoint and trapezoid estimates, 0.8632 and 0.8675.

3. Compute $\frac{d}{dx}\left(\int_x^1 \ln t\,dt\right)$.

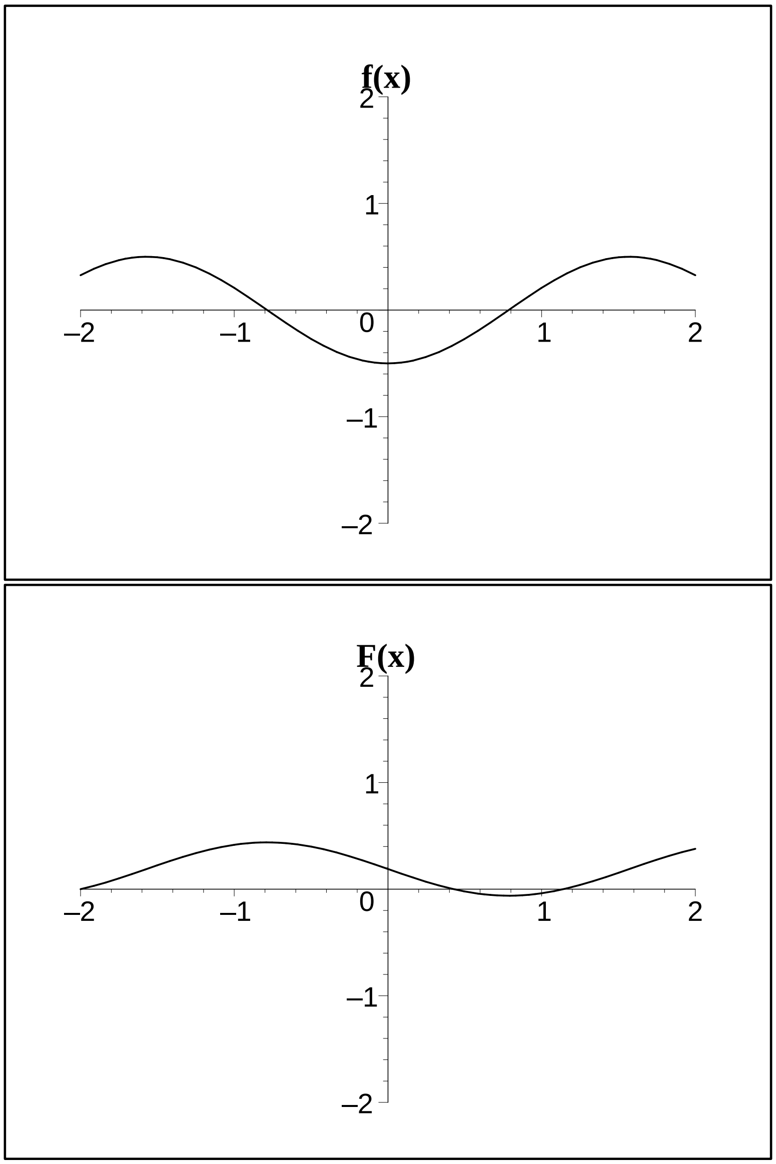
(This is problem 6.4.19 from your homework.) $\frac{d}{dx}\left(\int_x^1 \ln t\,dt\right) = \frac{d}{dx}\left(-\int_1^x \ln t\,dt\right) = -\ln x$, **by the Fundamental Theorem of Calculus.**

4. The graph of $f(x)$ is given below. Let $F'(x) = f(x)$. **Please don't get $\boldsymbol{F}$ and $\boldsymbol{f}$ mixed up while doing this problem!**

(a) What are the critical points of $F(x)$?

(b) Which critical points are local maxima, which are local minima, and which are neither?

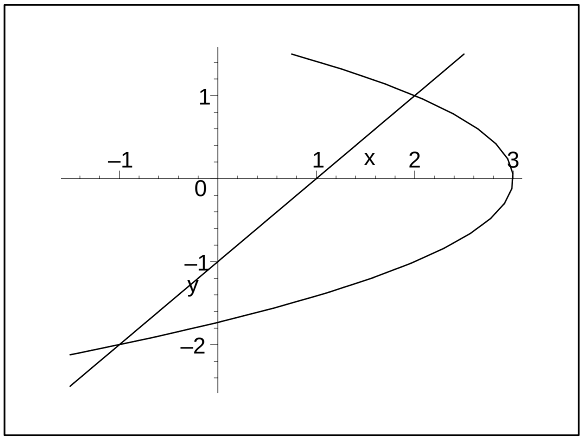
(c) Sketch the graph of $F(x)$ on the bottom axes, given that $F(-2) = 0$.



(a) The critical points are where the derivative $f(x)$ is zero, so roughly at $x = -0.8$ and $x = 0.8$.

(b) The derivative $f(x)$ goes from positive to negative at -0.8, so that's a maximum. The derivative goes from negative to positive at 0.8, so that's a minimum.

5. Find the area of the region bounded by the graphs of $x = 3 - y^2$ and $x = y + 1$. (The graphs intersect at the points $(-1, -2)$ and $(2, 1)$, as shown in the figure.)



Use horizontal slices. The width of a slice is $(3 - y^2) - (y + 1) = 2 - y^2 - y$**, so the total area is** $\int_{-2}^{1} (2 - y^2 - y)\, dy = 9/2$**.**