(1) (20 pts) Find the maximum and minimum values of the function $f(x) = -x^{2/3}$ on the closed interval $[-8, 8]$.

Find the critical pts.

$$f'(x) = -\frac{2}{3}x^{-\frac{1}{3}} = \frac{-2}{3\sqrt[3]{x}}.$$ This doesn't exist at $x=0$,

and is never 0, so $x=0$ is the only critical point.

So, check the values at 0, -8, and 8.

crit. pt. end pts.

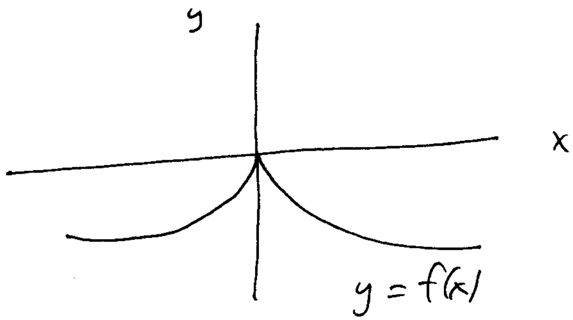
$f(0) = 0$

$f(8) = -\left(\sqrt[3]{8}\right)^2 = -(2)^2 = -4$

$f(-8) = -\left(\sqrt[3]{-8}\right)^2 = -(-2)^2 = -4$

So the max. value is 0, min. value is -4.

Picture



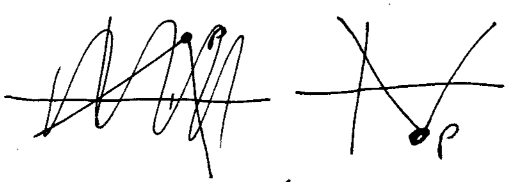
(2) (16 pts) Let $f(x)$ be a continuous function. Label each of the following statements as true or false; if a statement is false, give an example showing that it's false.

(a) If x_0 is not a critical point of $f(x)$, then x_0 is not a local maximum for $f(x)$.

True (every local max. is a crit. pt.)

(b) If x_0 is not a local maximum of $f(x)$, then x_0 is not a critical point for $f(x)$.

False:

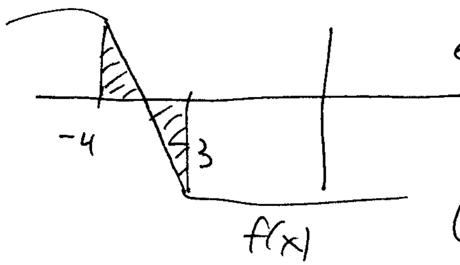


P isn't a local max, but it is a crit. pt.

(lots of possible answers)

(c) If $\int_{-4}^{-3} f(x)\,dx$ equals 0, then $f(x) \equiv 0$ (i.e., $f(x)$ is the constant function 0).

False:



equal areas above & below the x-axis

(lots of possible answers)

(d) If y lies on the curve $x^2 + 5y^2 = 1$, then dy/dx is defined at every point on the curve.

False: Find $\frac{dy}{dx}$ by implicit diff: $2x + 5y\frac{dy}{dx} = 0$, or

$\frac{dy}{dx} = \frac{-2x}{5y}$. This is undefined when $y = 0$, e.g., at the point $(1,0)$ or $(-1,0)$.

(3) (25 pts) Reliable Red Barclay is a truck driver. Suppose that it costs Red $\left(1 + (\frac{2}{30{,}000})v^{3/2}\right)$ dollars per kilometer to operate his truck at v kilometers per hour. If there are additional costs (for things like caffeine pills) of \$10 per hour, how fast should he drive to minimize the total cost of a 1000 kilometer trip? (You should ignore speed limits- Red does.)

Minimize total cost = (cost of operating truck) + (cost of pills)

= (operating cost per km)·(1000 km) + (\$10 per hour)·(# of hours)

If Red goes v kph., it will take him $\frac{1000}{v}$ hrs to travel the 1000 miles. Thus the total cost is

$$C(v) = \left(1 + \frac{2}{30{,}000}v^{3/2}\right)(1000) + 10 \cdot \frac{1000}{v}$$

The endpoints are $v = 0$ (minimum speed) and $v = \infty$ (max. speed - Red has a fast truck).

Find the critical pts: $C'(v) = \frac{1}{10}v^{1/2} - \frac{10{,}000}{v^2}$

This doesn't exist when $v = 0$. To find other critical pts, set $C'(v) = 0$,

or $\frac{1}{10}v^{1/2} = \frac{10{,}000}{v^2}$, $v^{5/2} = 100{,}000$, $v = (100{,}000)^{2/5} = \left(\sqrt[5]{100{,}000}\right)^2$

$= 10^2 = 100.$

So, check $v = 0, \infty, 100$.

$C(0)$: $\lim_{v \to 0} C(v) = \infty$

$C(\infty)$: $\lim_{v \to \infty} C(v) = \infty$

So the minimum cost must be at speed $v = 100$ kilometers/hr.

(4) (12 pts) Compute the following limits.

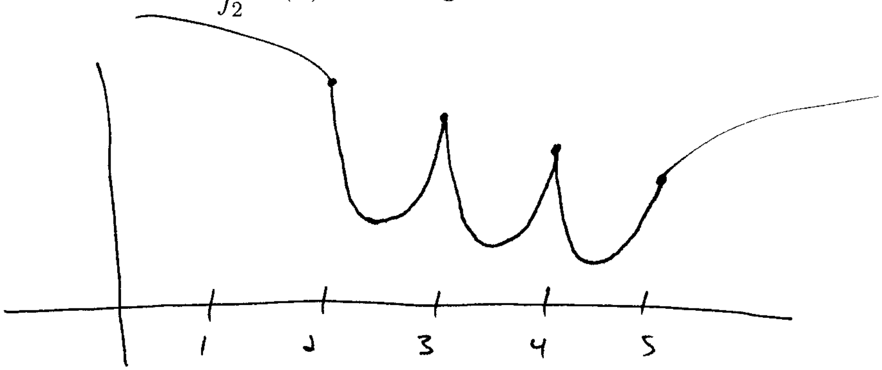
(a) $\lim_{x \to 0} \frac{\ln x}{x^2}$ If we plug in, we get $\frac{-\infty}{0}$, so the limit doesn't exist - it blows up to $-\infty$.

(b) $\lim_{x \to \infty} \frac{\ln x}{x^2}$ Plug in & get $\frac{\infty}{\infty}$, so apply L'Hopital's rule:

$$\lim_{x \to \infty} \frac{\ln x}{x^2} = \lim_{x \to \infty} \frac{\frac{1}{x}}{2x} = \lim_{x \to \infty} \frac{1}{2x^2} = 0.$$

(5) (12 pts) Sketch the graph of a continuous function f (you don't need to give a formula for f) on the interval $[2, 5]$ with the property that with $n = 3$ subdivisions,

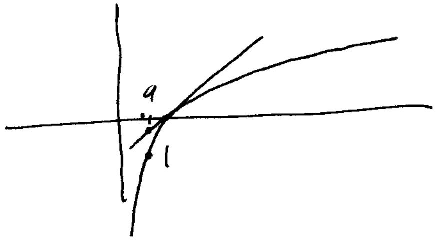
$$\int_2^5 f(x)\, dx < \text{Right-hand sum} < \text{Left-hand sum}.$$



(lots of possible answers)

(6) (15 pts) Use linear approximation to estimate $\ln(0.9)$.

$$f(x) \approx f(a) + f'(a)(x-a)$$



Use $a = 1$.

$\ln(1) = 0$. Since $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$, $\ln'(1) = 1$.

So $\ln(0.9) \approx 1 + 1(0.9 - 1) = 1 + (-0.1) = 0.9$.