Illinois Journal of Mathematics
Volume 48, Number 3, Fall 2004, Pages 1079–1080
S 0019-2082

# ADDENDUM TO "A FIXED POINT THEOREM FOR BOUNDED DYNAMICAL SYSTEMS"

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We say that a compact set $W$ is a *window* for a dynamical system (continuous map or flow) on $X$ if the forward orbit of every point $x \in X$ intersects $W$. If a dynamical system has a window then we will say that it is *bounded*.

In [6], we proved the following fixed point theorem.

THEOREM 1. *Every bounded dynamical system on $\mathbb{R}^n$ has a fixed point.*

Since that paper appeared, it has come to our attention that Theorem 1 had already been proved, in [2] in the case of maps and [9] in the case of flows. We wish to correct our oversight in this addendum.

Fournier proved the following:

THEOREM 2 ([2]). *If $X$ is an absolute neighborhood retract, $f : X \to X$ is a locally compact, bounded map, and the Lefschetz number of $f$ is nonzero, then $f$ has a fixed point.*

(A map is locally compact if every point has a neighborhood whose image is precompact.) Theorem 1 for maps is thus a special case of this result. In fact, the results in [2] go far beyond Theorem 2 and are concerned with finding very general circumstances in which the Lefschetz fixed point theorem applies. Further important work in this area appears in [1], [3], [4].

Srzednicki proved the following result for flows:

THEOREM 3 ([9]). *If $X$ is a Euclidean neighborhood retract and $\varphi$ is a bounded flow on $X$, then $X$ has the homotopy type of a compact polytope, and $\varphi$ has a fixed point provided that the Euler characteristic of $X$ is nonzero.*

Theorem 1 for flows is a special case of this result. Further results on bounded flows appear in [5].

Theorem 1, being less general than these two theorems, has at least the virtue of a much shorter proof. In addition, the outlook is rather different. The

Received April 28, 2004.
2000 *Mathematics Subject Classification.* Primary 54H25. Secondary 37B30, 37B25.

results in [2] and [9] are concerned with ways to generalize the applicability of topological fixed point theorems, and bounded dynamical systems appear as one way to do this; our work is concerned with the behavior of bounded systems, and the fixed point theorem appears as one aspect of this. Further results on bounded systems appear in [7], [8].

We regret our oversight in not referencing these earlier results in our original paper, and we thank Jan Andres, Lech Górniewicz, and Klaudiusz Wójcik for bringing them to our attention. We talked to researchers and spoke about our work for several months before our paper appeared, and no one was aware of the earlier work. Thus we hope that this addendum will bring that work some of the attention that it merits.

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