APPENDIX I

Reading Research Papers: Some Tricks of the Trade[2]

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Have you ever tried to read a book in a foreign language? Reading a research article or advanced textbook is much the same. You are working with a limited vocabulary, but you don't have unlimited time. It is as if the text were written in code; you want to find out what the message is, even if you can't decipher every word.

This essay describes a time-tested strategy for reading. I will assume you are reading a paper, but everything here also applies to chapters in books, lecture notes, and so on.

To start making progress, give up the notion that you must verify the truth of every single line, or that you must understand each paragraph before going on to the next. You may end up spending hours trying to understand a point that turns out to be unnecessary or unimportant. Worse yet, you may finally understand every line, only to find you have no idea what the paper is about as a whole.

The first step is to decide what parts of the paper to read carefully and what to skip. If the most important ideas are near the end of the paper it doesn't make sense to spend a lot of time worrying about the introduction. A good way to make this decision is to make several passes through the whole paper, without trying to follow all the details. On each pass, try to get more information about how the paper is organized, what's important, what's simple and what's difficult. The process is more like solving a puzzle than reading a novel. The type of information you are looking for is as follows.

What is this paper about?

What problem does the author address? Why is the problem interesting? What is the history of the problem? What are the results? (Results can include theorems, algorithms, analyses of experimental data, formulas, and so on.) Is the problem solved, or are only partial results given? What are the key steps leading to the results? A good paper will answer these questions explicitly; otherwise, you do your best to figure it out.

[2]This paper originally appeared as "Teaching Students to Read Papers" in *SIGACT News* 22(Fall 1991), 35-37.

What is new, interesting or important here?

What makes the paper worth reading? Is it the problem itself, the final results or solution, the method used, or some other feature? What are the authors' contributions and what builds on earlier work or standard methods? Sometimes it's not clear which results are important or new; you may have to consult someone more experienced, and you may revise your opinion later.

Which parts are worth spending time on?

Decide what parts of the paper you are going to try to understand in detail. Normally, these will be the derivations of the results you have previously identified as most important or most interesting. Be realistic about the amount you can read carefully; you can always add to your list later. You don't have to read things in the order given in the paper. I often work backwards, first trying to understand the major conclusions or steps, then working on the details (equations, subroutines, etc.) later. When stuck, I read whatever seems easy first, then work up to the more difficult parts.

Once you've zeroed in on the parts of the paper you most want to understand, the next step is to get down to details. You are not ready to do this until you have preliminary answers to the questions above (though you may revise your opinion as you read more carefully). Work through a small section at a time; don't try to understand everything at once.

Again, avoid the temptation to go through a section line by line. If you are not on the same wavelength as the author, you will find it very difficult to fill in any missing steps. You do not have to understand how each fact is derived. For example, authors often make side remarks for specialists in their field, which you can safely ignore. Even if the author claims that, "It is trivial to see that...", you should not take this literally. First read the paper as if the unproven claims are true. You can always try to prove them yourself later, if you have time after reading the important things. Don't worry if you can't prove them; they're only meant to be easy for someone who's an expert in the field.

On the other hand, if a fact labeled "obvious" is crucial for the results, it is worth spending the time trying to convince yourself that it is true. On rare occasions you may actually find a counter-example, proving that it is false, and revealing an important flaw in the paper that you would otherwise have missed.

When trying to understand a particular result in depth, the basic strategy is to "make it your own." A good method is first to spend a few minutes thinking about how you might approach the problem if you were trying to solve it. Don't spend hours doing this, just enough time so that you see what the difficulties are. Each time you get stuck, go back to the paper for another hint. Once you know the answer, go back and try to explain it in your own words. If you come up with a different way of solving the problem, so much the better. Whether or not you solve the problem you will find it much easier to understand the solution after having experience with the problem.

Don't look things up until you are sure you have to. If there are words you don't know, try to read around them. Sometimes the author will give a reference to another paper or book. You can often understand what's going on without reading the reference. Most references are there only to give proper credit for ideas or results which are not the author's own. Avoid searching through the library until it becomes absolutely necessary. Even if the paper is laden with unfamiliar jargon, do not give up too soon. Often, things that seem incomprehensible at first become clearer, once you realize that much can be ignored.

If you get stuck on the part you're reading, put it aside and try a different part of the paper. Staring at the same lines over and over is frustrating and pointless. Of course, even after you have done all of this work, there will be things you can't make sense of. Make a list of your questions and consult a friend or faculty member. You may also find that by explaining the problem to someone else you understand it more clearly.

Probably the hardest part is deciding when you should ask for help and when you should try harder on your own. As a general guide, if your question is of the form "I can't understand any of this..." you haven't done enough work yet. You're ready to ask when you understand enough to ask specific questions, such as "Where did this formula come from?", "Is there an example for this definition?", or "How is this algorithm related to that one?".

If you get ambitious, try thinking about the open problems that are mentioned (usually at the end of the paper). "Open" means that no one has published a solution yet. You probably won't find a complete solution, but you may discover some interesting results.

The process I have described is time-consuming. However, it is a much better use of your time than reading by rote, as it gives you a better grasp of the point of the paper, the importance of the results, and the choice of particular methods. Reading is an art that takes patience and practice; when you have learned it, you are capable of understanding a lot more than you thought you could.

APPENDIX II
Giving Oral Presentations in Mathematics

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These notes will guide you through giving a seminar talk, whether short or long, formal or informal. After you graduate, you will find yourself in many situations where it is important to appear professional and self-confident despite nervousness and uncertainty. You will find the experience of planning a talk to be invaluable later, if you make the effort to do it well now. It is normal in public speaking to get stage fright, develop nausea, or go blank (whatever your nightmare is); when you are well-prepared, you can give a successful performance anyway.

Even experienced speakers know that a lot of preparation is required to give a concise and interesting talk. There is a wonderful quote, attributed to Abraham Lincoln: "If you want me to give a ten minute speech, give me two weeks.... If you want a two hour speech, I'm ready now!"

I. What to Include in a Talk

A well-constructed talk has a clear focus. For example, you might present and prove one important theorem, or solve a single interesting problem, or outline a few important results in a single area. Tell the audience enough to hold their interest and to teach them something new, but avoid confusing them with a barrage of detail.

Right at the beginning, state the topic or problem you're going to discuss. State the conclusions or results as early as possible, as well as at the end of the talk. Make sure the audience knows where you are headed before you launch in to the details.

Motivate the results you present; let the audience know why the results are interesting or important. For example, give a brief history or an application of a theorem, or show how a problem is related to another well-known problem.

Only give a careful derivation or proof if it will help the audience understand or appreciate a result. It is often better to give only the key ideas rather than a full proof. For example, explaining briefly that an equation can be solved by elementary algebra can be more satisfying than seeing the solution step-by-step. If you are explaining an algorithm, illustrating it with an example can be clearer than writing it out in words.

Keep jargon and symbols to an absolute minimum. Only introduce terms or definitions that you actually use. Remind the audience of the meaning of a new term each time you use it.

Be sure that your talk fits the given time period by timing the talk in advance. Plan the talk so that you can omit or add sections when the talk goes faster or slower than you anticipated. If you are famous or have a spellbinding lecture you may be forgiven for exceeding the time limit; otherwise it is considered rude.

II. Preparing a Talk Step-by-Step

1. Write down ideas, ignoring organization and specific wording at first. It is often easier to write short sections rather than trying to write the whole talk at once.

2. Organize the material you have and prepare an outline. If you will be using a chalkboard, write down the things you want to write on the board. If you are making transparencies, work them out roughly on paper; write very large, and don't put any more on each sheet than is necessary.

3. Start over (no kidding)! Decide what is most important; throw away everything else, and rewrite the talk. Pare down what you write on the board or the transparencies to what is absolutely essential. Be merciless: if it does not convey information, leave it out.

4. Prepare what you will say during and between transparencies, or after writing on the chalkboard.

5. Practice and *time* the talk. Revise the talk for length if necessary. Don't talk faster, make the talk shorter! Write up a final draft of your lecture notes, and draw final versions of slides on paper.

6. Get constructive criticism from someone experienced (such as your instructor). Generally, you will find people willing and able to help you only after you have made the effort to prepare a good draft. Do not get upset if you are told to start over or make major revisions, no one makes substantial suggestions unless they care about your performance.

7. Revise the talk once more. If using transparencies, transfer your sketches from paper to transparencies. If necessary, rewrite your notes so that you can read them easily; write down reminders to yourself

(such as "pause", "check time", or "slow down here"). Number your notes and transparencies so you can reorganize them if you drop them.

8. Dress rehearsal: find a private place to practice the talk in full, and time it again. Save some energy for the actual talk, however; too much practice will make a talk sound mechanical.

9. The Real Thing.

10. Celebrate!

This may seem like a lot of work for a short presentation, but preparation and practice are the secret to appearing knowledgeable and composed. Giving a talk requires rehearsal, just like performing music or acting. The writing and rewriting not only improves the talk, but helps you to better understand your topic. If you care about your audience, and want them to follow your talk, you will find that the work pays off.

III. Looking Professional

The primary rule in giving a good talk is to be considerate of your audience. It is surprisingly easy to lose the audience, so keep things simple. The audience would rather understand your talk than be impressed by your extensive knowledge.

Talk only about what you know; if you don't understand it, the audience won't either.

Illuminate results rather than just stating or verifying them. A well-chosen, elementary example is often better than a formal proof.

Avoid messy arithmetic unless it is the point of the talk. You do not have to show all intermediate steps for every derivation. Few listeners will get upset if you say, "and then, by a tedious but straightforward calculation, which I will omit, we get..."

A talk that begins, 'let me get some definitions out of the way...,' followed by three slides covered with tiny writing, is deadly for an audience. It is like trying to follow a conversation in a new language after only one glance at a dictionary. Only define a new term if you plan to use it immediately. Give simple examples, or pictures to illustrate definitions. Briefly remind the audience of the definition each time you use it.

The audience usually doesn't start concentrating until there is a visual cue, such as a transparency, and may miss that which is only spoken. A good general strategy is to 'show' first and then 'tell'; that way the audience has two chances to get the point. However, choose visual aids carefully; make sure they are easy to see and as simple as possible. Be creative, but don't exhaust the audience with special effects.

If you want to give an argument step-by-step, a chalkboard is useful, since you can refer back to definitions or earlier steps. Also, writing on the board can help you slow your pace. If you have a complex figure or chart, which takes too long to draw on the board, use transparencies or photographic slides. If you use transparencies alone, organize your talk so that one does not have to remember a previous slide. It is difficult to retrieve a transparency after it has been shown, and people have short memories. If you use a physical model, make sure it is large enough.

Write large and legibly, and only write down what is necessary. Repeat and explain everything that you write. Leave transparencies up as long as possible, so people have time to read them.

Let the audience know when you are about to finish the talk; conclude, don't just stop talking. For example, summarize what you have said, or discuss the questions left unanswered by the work presented.

Have extra material prepared and decide in advance what you can omit. Anticipate questions, and prepare answers in advance. It is hard to think when you are nervous.

Turn up your volume. Talk to the people at the back of the room. Make eye contact, or at least lift your head and face the audience; this makes your voice more audible and animated.

Get rid of potential distractions like keys or bracelets. Expect to sweat and get chalk on your clothes, and dress accordingly. Bring extra chalk, or pens for transparencies. Bring whatever you need to be comfortable, such as a cup of water or tissues.

No matter how terrible you feel, the audience can't tell that your mouth is dry, your palms are sweating, and your knees are shaking—so relax.

APPENDIX III
Standard Visual Aids for Technical Presentations

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Using a Chalkboard

If you have never used a chalkboard before, or have a difficult figure to draw, practice in advance. Write large and press hard. If you are left-handed, practice writing until you can write from left to right without accidentally erasing what you've written.

Erase the entire board before you begin. Make sure you have enough chalk before you start. If you want to use colored chalk, bring your own. If the chalk squeaks or vibrates, break it in half.

Give the audience as much time as possible to read and absorb what is on the board. Don't erase anything until absolutely necessary. After you write something down, step away from the board so everyone can see. Avoid talking while facing the board. Repeat what you have written after it is complete.

Most people like to fill up the sections of the board from left to right, so you can read them like pages in a book. Try not to write across the vertical lines in the board; many people find this disturbing. Don't bother to correct small errors, and don't worry about making the writing look perfect; this can slow your pace enough to be distracting.

Using Transparencies

Transparencies for an overhead projector (8 1/2" by 11"), also called slides, foils, or viewgraphs, are common in scientific presentations. You can write on them or transfer figures directly using a copy machine. Transparencies and pens are usually available at office or art supply stores.

For the greatest impact, use relatively few slides, with simple, carefully chosen images. Everything on a slide should be essential–if not, get rid of it. Put only what is *logically* necessary on a single slide, not what is physically possible.

You can use different colors for emphasis or to make a picture clearer. However, changing colors randomly or using too many colors can be distracting.

The audience will be looking at the transparencies from across the room, looking over peoples' heads. Write very large; four or five words to a line is about right, using all capital letters. If you prepare your transparencies with a word processor, use a *very* large font. Leave space between lines. Leave at least a one inch margin top and bottom, to avoid having parts cut off (most overhead projectors have a square platform). If you have figures or pictures, check that the images are visible from far away.

After you put a slide down, step away from the projector so the audience can see it. Look at the screen to check the picture. To draw attention to part of a slide, point to the screen with a long pointer or pen (with the cap on!). If this is awkward, lay a pen down on the slide pointing in the right direction. Hands tend to shake, so avoid using your finger as a pointer. Holding on to the slide while talking makes it wiggle around, and makes it hard to read. If your slides curl up or blow away, weight them down with coins or pens.

Leave slides up as long as possible; many people read slowly. If there are words on a slide, it's a good idea to repeat them all, in case not everyone can see all the words, and so the audience does not worry that they have missed something.

You can overlay transparencies to show a construction or algorithm in stages. If you overlay one or more transparencies, it is a good idea to tape them together into a "book"; it's hard to line up figures when you're nervous! Slides are not perfectly transparent, so if you stack them, make sure you can still see through the stack.

The best all-purpose slides seem to be those for regular copiers; less expensive types tend to curl or stick together; some are covered with a film and cannot be erased. If the talk is to be given more than once, you should use "permanent" pens. These usually can be erased with alcohol (experiment first). I have had good results with both *Stabilo* pens and *Sharpie* laundry markers. Water-soluble pens are easier to erase, and are good for writing on a slide during a talk, but run easily.

When writing on slides, keep a piece of paper under your hand to prevent smearing. Avoid handling the slides; protect the faces by putting sheets of paper between them.

A GENTLE INTRODUCTION TO INFINITE SERIES USING A GRAPHING CALCULATOR

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ABSTRACT: This article contains concrete suggestions for using a graphing calculator to enhance first-year calculus students' understanding of infinite series. For many students, this topic is the first abstract mathematical notion encountered and as such, represents a not-entirely-trivial hurdle. The techniques described herein represent an alternate, useful approach without the need for expensive computers.

KEYWORDS: Infinite series, graphing calculator, mathematics education, student understanding.

INTRODUCTION

Oh, no—not again! Several years ago, I was in the middle of grading a set of second–semester calculus tests, the first ones covering infinite series. On more than one paper I found something like

$$\lim_{n\to\infty} a_{n+1}/a_n = 1/3,$$

so,

$$\sum_{n=1}^{\infty} a_n = 1/3.$$

It has been my experience that many times even good first-year calculus students have considerable difficulty when first encountering the notion of

*Author partially supported by NSF Grant No.USE8953948.