Math 81 PDES 3/4/2004 Homework Supplement

1. (a) Show that $\langle (a,b),(c,d)\rangle = ac + \frac{1}{2}(ad+bc) + bd$ is an inner product on $\mathbb{R}^2$.

(b) Show that $\mathcal{B} = \{\mathbf{x}_1 = (1,0), \mathbf{x}_2 = (1,-2)\}$ is an orthogonal basis for $\mathbb{R}^2$ with this inner product.

(c) Use this inner product to find the coordinates of the vector $(2,2)$ with respect to the basis $\mathcal{B}$. (You can probably do it just by looking at it, but humor me and use the inner product.)

2. Let $\mathbb{P}_2$ be the vector space of all real polynomials of degree less than or equal to two. For p and q in $\mathbb{P}_2$, define

$$\langle p,q\rangle := p(0)q(0) + p(1)q(1) + p(5)q(5).$$

Show that this defines an inner product on $\mathbb{P}_2$. What is the "length" of the polynomial $2x^2 + x$ with this inner product?