Math 6D Homework #3
Due in class Friday, Nov. 21.

1. Let $f(x)$ be a differentiable function, and let $N(x)$ be its Newton function.
(a) Calculate $N'(x)$.
(b) Show that every point x_0 such that $f(x_0) = 0$ and $f'(x_0) \neq 0$ is an attracting fixed point for $N(x)$. What does this say about how well Newton's method works?
(c) Recall from your exam that the Babylonian function $B(x)$ for finding the square root of a positive number a is $B(x) = \frac{x+\frac{a}{x}}{2}$. Show that $B(x)$ is the same as the Newton function for $f(x) = x^2 - a$.

2. Recall from the practice midterm the doubling map $G : [0,1] \to [0,1]$ defined by

$$G(x) = \begin{cases} 2x & \text{if } 0 \leq x < 1/2, \\ 2x - 1 & \text{if } 1/2 \leq x \leq 1. \end{cases}$$

(We can also define G as $G(x) = 2x \mod 1$.) Let L denote the subinterval $[0, 1/2]$ and R the subinterval $[1/2, 1]$.
(a) Calculate and sketch the intervals LL, LR, RL, RR and $LLL, LLR, LRL, LRR, RLL, RLR, RRL, RRR$. What is the size of an interval whose name has k letters?
(b) Draw the transition graph for G.

3. Recall that the tent map $T : [0,1] \to [0,1]$ is defined by

$$T(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1/2 \\ 2 - 2x & \text{if } 1/2 \leq x \leq 1. \end{cases}$$

Repeat parts (a) and (b) of #2 for T.

4. Define a function $H : [0,1] \to [0,1]$ by setting

$$H(x) = \begin{cases} -x + 1/3 & \text{if } 0 \leq x \leq 1/3 \\ 3x - 1 & \text{if } 1/3 \leq x \leq 2/3 \\ -2x + 7/3 & \text{if } 2/3 \leq x \leq 1. \end{cases}$$

Let L denote the subinterval $[0, 1/3]$, M the subinterval $[1/3, 2/3]$, and R the subinterval $[2/3, 1]$.
(a) Calculate and sketch the intervals $LL, LM, LR, ML, MM, MR, RL, RM, RR$. What can you say about the size of an interval whose name has k letters in it? (This is more complicated than it was in #2 and #3.)
(b) Draw the transition graph for H.