Math 6D Homework #2
Due in class Friday, Nov. 7.

1. Use graphical analysis to describe the fate of all orbits for each of the following functions. Use different colors or different diagrams for orbits that behave differently.
   (a) $F(x) = 2x$
   (b) $F(x) = \frac{1}{3}x$
   (c) $F(x) = -2x + 1$
   (d) $F(x) = x^2$
   (e) $F(x) = -x^3$

2. Consider $F(x) = x^2 - 1.1$. First find the fixed points of F. Then use the fact that these points are also solutions of $F^2(x) = x$ to find the two-cycle for F.

3. Find a period-two orbit for the tent map

$$T(x) = \begin{cases} 2x & \text{if } 0 \le x \le 1/2 \\ 2 - 2x & \text{if } 1/2 \le x \le 1. \end{cases}$$

4. We're going to use Newton's method to solve the equation $x^2 + 1 = 0$.
   (a) Why is this a stupid thing to do?
   (b) Despite that, find the Newton function $N(x)$.
   (c) Show that if the initial guess is $x_0 = 1/\sqrt{3}$, then the subsequent guesses alternate between $-1/\sqrt{3}$ and $1/\sqrt{3}$ (no decimal approximations, please).

5. Use graphical analysis to describe completely all orbits of the associated Newton function for each of the following functions.
   (a) $F(x) = 4 - 2x$
   (b) $F(x) = x^2 - 2x$
   (c) $F(x) = 1/x$

6. Let F be a one-to-one differentiable map of the real line to itself. One-to-one means that if $F(x) = F(y)$, then $x = y$. F is called *increasing* if $x < y$ implies $F(x) < F(y)$, and *decreasing* if $x < y$ implies $F(x) > F(y)$.
   (a) Show that either F is increasing everywhere or F is decreasing everywhere.
   (b) Show that every orbit $\{x_0, x_1, x_2, \ldots\}$ of F^2 satisfies either $x_0 \ge x_1 \ge x_2 \ge \ldots$ or $x_0 \le x_1 \le x_2 \le \ldots$. (HINT: Show that $(F^2)'(x) \ge 0$ for all x.)
   (c) Show that every orbit of F^2 either diverges to $+\infty$ or $-\infty$ or converges to a fixed point of F^2.
   (d) What does this imply about convergence of the orbits of F?
   (e) Give examples of some functions displaying some of the different behavior discussed above.