

Math 6D Homework #1  
Due in class Friday, Oct. 31.

1. Let  $F(x) = x^2 + 1$ . Compute the first five points on the orbit of 0.
2. Let  $G(x) = x^2 - 2$ . Compute  $G^2(x)$  and  $G^3(x)$ .
3. Let  $H(x) = |x|$ . Compute  $H^2(x)$  and  $H^3(x)$ . What are the eventually fixed points for  $H$ ?
4. Find all real fixed points (if any) of the following functions:
  - (a)  $F(x) = 3x + 2$
  - (b)  $F(x) = x^2 + 1$
  - (c)  $F(x) = |x|$
  - (d)  $F(x) = x \sin x$
5. Let  $F(x) = 1 - x^2$ . Show that 0 is a period-2 point for  $F$ .
6. Consider the function  $G(x) = |x - 2|$ .
  - (a) What are the fixed points of  $G$ ?
  - (b) If  $m$  is an odd integer, what can you say about the orbit of  $m$ ?
  - (c) If  $m$  is an even integer, what can you say about the orbit of  $m$ ?
7. Consider the *tent map*  $T : [0, 1] \rightarrow [0, 1]$ , defined by

$$T(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1/2 \\ 2 - 2x & \text{if } 1/2 \leq x \leq 1. \end{cases}$$

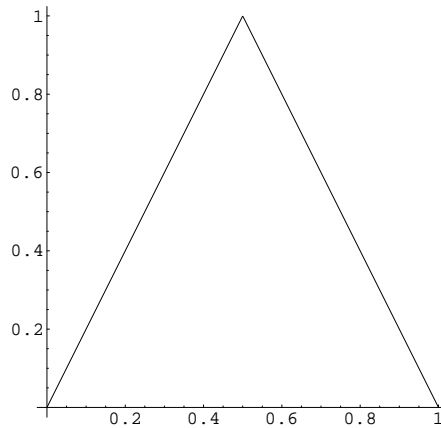


FIGURE 1. The tent map.

- (a) Find a formula for  $T^2$ , and sketch its graph.
- (b) Find all fixed points for  $T$  and  $T^2$ .
- (c) Find a formula for  $T^3$ , and sketch its graph.
- (d) What does the graph of  $T^n$  look like?