Math 6D Practice Final

1. Is 1/30 in the middle-thirds Cantor set K? (HINT: Remember that K is the set of all points that stay in the interval $[0,1]$ under all iterates of the slope-3 tent map.)

2. Show that any subset of a countable set is itself countable.

3. Compute the box-counting dimension of the Sierpinski carpet. (The Sierpinski carpet, named after Waclaw Sierpinski, is a fractal derived from a square by cutting it into 9 equal squares with a 3-by-3 grid, removing the central piece and then applying the same procedure ad infinitum to the remaining 8 squares. See the figure below.)

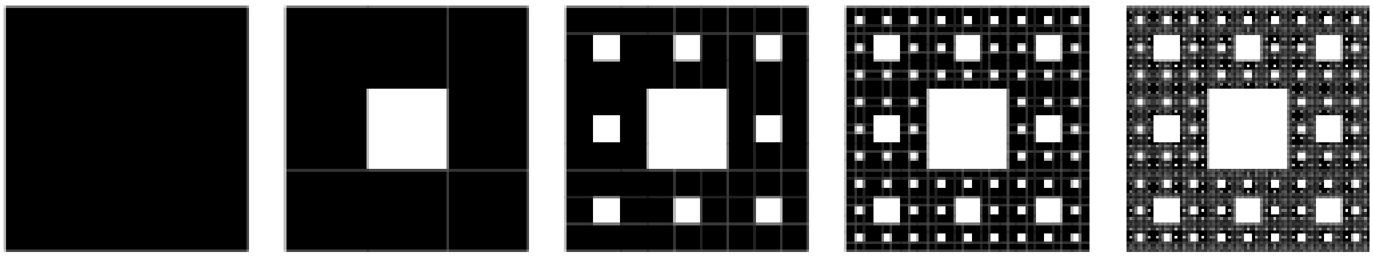

FIGURE 1. The Sierpinski carpet

4. Show that the map $G : [0,1] \to [0,1]$ given by $G(x) = 2.5x(1-x)$ is not chaotic.

5. Assume that $a < b < c < d$ are points on the real line, and that F is a continuous map satisfying $F(a) = b$, $F(b) = d$, $F(c) = a$, and $F(d) = c$. For simplicity, assume that F is monotonic (increasing or decreasing) except possibly at the four points mentioned.

(a) Sketch the graph of F.

(b) Draw the transition diagram for F.

(c) What periods must exist? (NOTE: You don't need to memorize the Sharkovskii ordering. I'll give it to you on the exam if you need it.)

6. The *dyadic rationals* are the real numbers of the form $\dfrac{m}{2^n}$, where m and n are integers. Are the dyadic rationals dense in the real line $\mathbb{R}$?

7. Explain what it means for a point to be sensitive.

8. Determine whether the following points are in the Mandelbrot set.

(a) $c = 1/2$

(b) $c = i$