Math 6D Practice Exam Solns

(1) Look at orbit of $1/30$ under slope-3 tent map:

$$\frac{1}{30} \to \frac{1}{10} \to \frac{3}{10} \to \frac{9}{10} \to \frac{3}{10} \to \frac{9}{10} \to \dots$$

It does stay in $[0,1]$ forever, so it's in K.

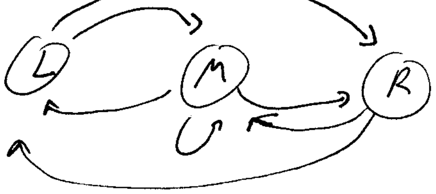
(2) Recall that a set is countable if it can be put in 1-1 correspondence with a subset of the natural numbers $\mathbb{N} = \{1, 2, 3, \dots\}$. If $S \subset T$, and T is countable, then S can be put in 1-1 correspondence with the subset of $\mathbb{N}$ that corresponds to the elts. of T that are also in S, so S is countable.

(3) Use boxes of side-length $\frac{1}{3^n}$ - need 8^n of them to cover. So the dimension is $\lim_{n\to\infty} \frac{\ln 8^n}{\ln 3^n} = \frac{\ln 8}{\ln 3}$.

(4) It can't be chaotic because the fixed pt. $3/5$ is a sink. That means that $3/5$ is not a sensitive pt. It also means that the periodic pts. aren't dense (because pts. near $3/5$ can't be periodic, since they get closer & closer to $3/5$), and that there can't be a dense orbit (because any orbit that comes near $3/5$ stays near, and so can't "fill up" the interval $[0,1]$).

(5) a)

b)



c) There's a period-3 pt (coming from the path $LMRL$, say), so there are points of all periods.

(6) Yes, they are dense. One way to see this is to realize that ~~any~~ the dyadic rationals are the set of points with a terminating binary expansion. Another way is to realize that ~~&~~ the intervals ~~of length~~ $\left[\frac{m}{2^n}, \frac{m+1}{2^n}\right]$ are getting smaller & smaller.

(7) See class notes.

(8) Look at the orbit of 0 under the map $z \mapsto z^2 + c$.

a) $0 \to \frac{1}{2} \to \left(\frac{1}{2}\right)^2 + \frac{1}{2} = 3/4 \to \frac{9}{16} + \frac{1}{2} = \frac{17}{16} \to > 3/4 \to > (3/4)^2 \to > \cdots \to \infty$, so $\frac{1}{2}$ is not in the Mandelbrot set

b) $i \to (i)^2 + i = -1+i \to (-1+i)^2 + i = -i \to -1+i \to -i \ldots$ eventually period-two, so i is in the Mandelbrot set.