1. Which of the following sets are dense in the interval $[0, 1]$? *Explain your reasoning.*
(a) The middle-third Cantor set K.

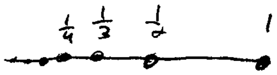
Not dense – K doesn't have any points inside $[1/3, 2/3]$

(b) The set of all numbers of the form $\frac{m}{10^n}$, where m and n are positive integers and $m < 10^n$.

Dense – this is the set of all numbers between 0 & 1 with terminating decimal expansions

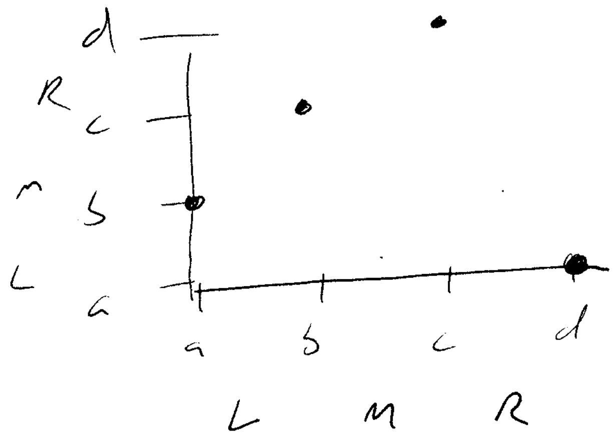
(c) The set of all numbers of the form $\frac{1}{n}$, where n is a positive integer.

Not dense –

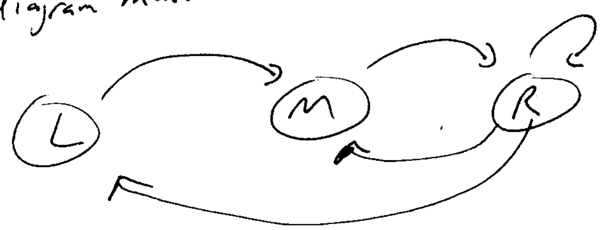


the set doesn't have any points between $\frac{1}{2}$ & 1.

2. Assume that $a < b < c < d$ are points on the real line, and that F is a continuous map satisfying $F(a) = b$, $F(b) = c$, $F(c) = d$, and $F(d) = a$. What periods must exist for F? *Explain your reasoning.*



The transition diagram must include at least these edges:

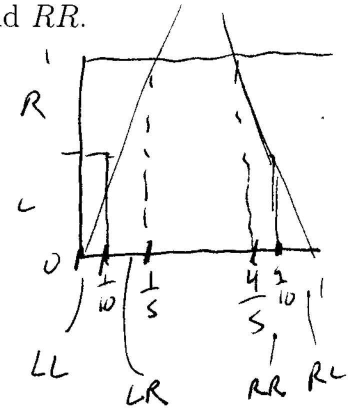


Since $\underline{LMRL}$ is a path, there is a fixed pt. x_0 for F^3 in LMR. x_0 can't be fixed, since no point is in L, M, & R simultaneously, so x_0 must have period 3. Thus F has periodic pts. of all orders.

3. Let $G : \mathbb{R} \to \mathbb{R}$ be the slope-5 tent map, given by

$$G(x) = \begin{cases} 5x & \text{if } 0 \le x \le 1/2 \\ 5 - 5x & \text{if } 1/2 \le x \le 1. \end{cases}$$

(a) Let L be the interval $[0, 1/2]$ and R the interval $[1/2, 1]$. Find and label the intervals LL, LR, RL, and RR.



So $LL = [0, \frac{1}{10}]$

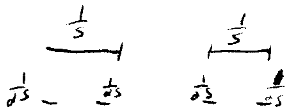
$LR = [\frac{1}{10}, \frac{1}{5}]$

$RR = [\frac{4}{5}, \frac{9}{10}]$

$RL = [\frac{9}{10}, 1]$

(b) Let S be the set of all points that stay in the interval $[0, 1]$ forever under G. What is the box-counting dimension of S?

S is the middle 3/5 Cantor set:



You need 2^n intervals of length $\frac{1}{5^n}$ to cover S, so the box-counting dimension is $\lim_{n\to\infty} \frac{\ln 2^n}{\ln 5^n} = \lim_{n\to\infty} \frac{n \ln 2}{n \ln 5} = \frac{\ln 2}{\ln 5}$

(c) Let T be the set of all points that stay in the interval $L = [0, 1/2]$ forever under G. What is the box-counting dimension of T?

0 is the only pt. that stays in L forever, and it has box-counting dimension zero.

4. Give an example of a dynamical system that you've encountered outside of this class. Does it exhibit chaotic behavior? Explain.

Lots of possible answers.