

6B HW #4 Solns

1) a) No b) YES ($= 2 + (x-1) + (x-1)^2 + \dots$) c) YES

2) a) $R = \lim_{n \rightarrow \infty} \left| \frac{(-1)^n}{n+1} \cdot \frac{n+2}{(-1)^{n+1}} \right| = \lim_{n \rightarrow \infty} \left| \frac{n+2}{n+1} \right| = 1$. Check endpoints: $1 \notin -1$

$x=1$: ~~$\sum_{n=1}^{\infty}$~~ $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$ converges

$x=-1$: $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$ diverges.

So int. of conv. is ~~\mathbb{R}~~ $(-1, 1]$.

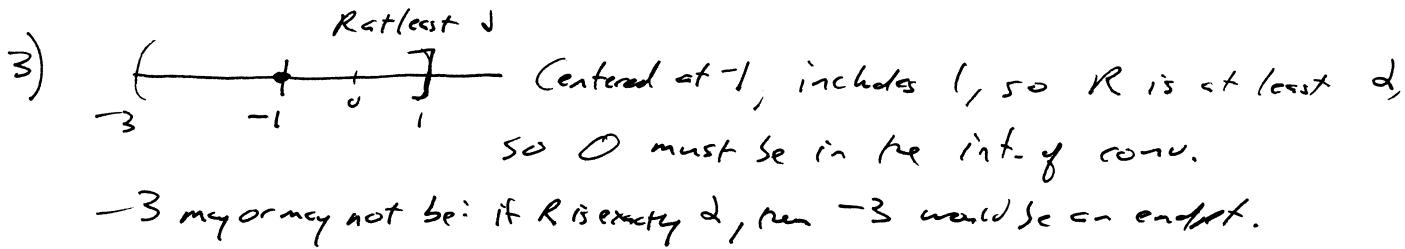
b) $= \sum \left(\frac{-1}{n} \right)^n x^n$. $R = \lim_{n \rightarrow \infty} \left| \frac{\left(\frac{-1}{n} \right)^n}{\left(\frac{-1}{n+1} \right)^{n+1}} \right| = \lim_{n \rightarrow \infty} \frac{(n+1)^{n+1}}{n^n} = \lim_{n \rightarrow \infty} (n+1) \cdot \left(\frac{n+1}{n} \right)^n = \infty$

So int. of conv. is whole line - converges for every x .

c) $= \sum n^2 x^n$. $R = \lim_{n \rightarrow \infty} \frac{n^2}{(n+1)^2} = \lim_{n \rightarrow \infty} \frac{1}{n^{2-2n+1}} = 1$, so $R=1$.

Check endpoints: $1 \notin -1$. $x=1$, set $1+4+9+\dots$ -diverges

$x=-1$, set $-1+4-9+16-\dots$ -diverges. So int. of conv. is $(-1, 1)$.



4) Can't use ratio test b/c half the time you'd be dividing by 0 (all the odd coefficients are 0). Int. of conv. for $1 - \frac{t}{3} + \frac{t^3}{9} - \dots$ is ~~\mathbb{R}~~ $-3 < t < 3$. Plug in x^t for t , set $-3 < x^2 < 3$, or $x^2 < 3$, or $-\sqrt{3} < x < \sqrt{3}$.

5) Do it termwise, set $\frac{x^2}{2} - \frac{x^4}{4!} + \frac{x^6}{6!} - \frac{x^8}{8!} + \dots$

- 6) a) no - has sines, not powers of x
 b) compare to $\sum \frac{1}{n!} x^n$
 c) at $x=0$, set $\frac{\cos 0}{1} + \frac{\cos 0}{2!} + \frac{\cos 0}{3!} + \dots = 1 + \frac{1}{2} + \frac{1}{3!} + \dots \rightarrow \infty$

- 7) Mac series: $f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \dots$
 $f(x) = \ln(1-x)$ $f'(x) = \frac{-1}{1-x}$, $f''(x) = \frac{-1}{(1-x)^2}$, $f'''(x) = \frac{-2!}{(1-x)^3}$, $f^{(4)}(x) = \frac{-3!}{(1-x)^4}$
 $f(0) = 0$ $f'(0) = -1$ $f''(0) = -2!$ $f'''(0) = -3!$... $f^{(n)}(0) = -n!$
 So the series is $0 - x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4 - \dots = \sum_{n=1}^{\infty} -\frac{1}{n}x^n$
 Int. of conv. is $(-1, 1)$.

- 8) $e^x = 1 + x + \frac{x^2}{2!} + \dots$, so $c^2 = 1 + \frac{4}{2!} + \dots$
 9) $1 + x + x^2 + \dots = \frac{1}{1-x}$ (if $|x| < 1$). So we $\frac{1}{1-x} = 5$, $1 = 5 - 5x$, $5x = 4$, $x = \frac{4}{5}$

- 10) Taylor series is $f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$
 So the coeff. of x^n in the series is $\frac{1}{n!} \cdot f^{(n)}(0)$.
 Hence, $C_n = \frac{1}{n!}$.
 So, $C_1 = \frac{1}{1} = \frac{1}{1} \cdot f'(0)$, so $f'(0) = 1$
 $C_2 = \frac{1}{2} = \frac{1}{2!} \cdot f''(0)$, so $f''(0) = 1$
 $C_3 = \frac{1}{3} = \frac{1}{3!} f'''(0)$, so $f'''(0) = \frac{\frac{1}{3}}{\frac{1}{3!}} = 2$
 $C_{10} = \frac{1}{10} = \frac{1}{10!} f^{(10)}(0)$, so $f^{(10)}(0) = \frac{10!}{10} = 9!$