Math 6B HW #2 Solns

1.1.13 a) sometimes true b) always true (bdd. by a_0 & 0)
c) sometimes true d) sometimes true e) always true f) sometimes true
n) always true: $a_n \to 0$

1.2.1 a) geometric, $\to \frac{1}{1-x}$ if $|x|<1$
b) not geometric c) geometric ($a_i = \frac{1}{2^i}\cdot 2$), sum $= 4$
d) geometric ($r = -\frac{1}{3}$), $\to \frac{1}{1-\frac{1}{3}} = 3/4$
e) geometric ($r = \frac{1}{1-x}$), converges if $x>2$ or $x<0$, to $\frac{x-1}{x}$

1.3.3 $M = \dfrac{160{,}000\,(.08)\,(1.08)^{360}}{(1.08)^{360}-1}$

1.3.5 $h + 2rh + 2r^2h + \dots = h + 2rh\,(1 + r + r^2 + \dots) = h + \frac{2rh}{1-r}$

1.3.6 $\frac{\sqrt{h}}{4} + \frac{\sqrt{rh}}{2} + \frac{\sqrt{r^2h}}{2} + \dots = \frac{\sqrt{h}}{4} + \frac{\sqrt{rh}}{2}\left(1 + \sqrt{r} + (\sqrt{r})^2 + \dots\right) = \frac{\sqrt{h}}{4} + \frac{\sqrt{rh}}{2}\cdot\frac{1}{1-\sqrt{r}}$

2.1.3 Does still converge, but could be greater than Σb_n

2.1.4 a) converges (compare to $1/3 + 1/3^2 + 1/3^3 + \dots$)
b) converges if $x^2 < 3$ (compare to $x^2/3 + x^4/3^2 + \dots$)
c) converges (compare to $\frac{3}{4} + \left(\frac{3}{4}\right)^2 + \dots$)
d) converges as long as $x^2 < 2$
e) converges (compare to $\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}^2} + \dots$)

2 a) $\sum_{n=1}^{\infty} \frac{1}{n^2}$ b) $\sum_{n=1}^{\infty} \frac{2^n - 1}{2^{n+1}}$ c) $\sum_{n=1}^{\infty} \frac{1}{n!}$ (other answers possible)

3 $\frac{1}{3}\cdot\frac{1-\left(\frac{1}{3}\right)^5}{1-1/3} = \frac{323}{648}$

4 a) $\frac{1}{1-1/3} = \frac{3}{2}$ b) $\frac{1}{1-(-1/3)} = 3/4$ c) diverges: $r = 1.01 > 1$

(5) a) $.8\overline{8} = \frac{8}{10} + \frac{8}{100} + \frac{8}{1000} = \frac{8/10}{1-\frac{1}{10}} = 8/9$

b) $.\overline{36} = \frac{36}{100} + \frac{36}{10{,}000} + \dots = \frac{36}{100} \cdot \left(\frac{1}{1-1/100}\right) = 36/99 = 4/11$

(6) $1\text{ bn.} + .75(1\text{ bn.}) + (.75)^2(1\text{ bn.}) + \dots = \frac{1\text{ bn}}{1-.75} = 4\text{ bn.}$

(7) $-1-2i$

(8) $|z| = \sqrt{169} = 13$, $|z^2| = |z|^2 = 169$, $\left|\frac{1}{z}\right| = \frac{1}{|z|} = 1/13$

$|z+\bar{z}| = |-5+12i + -5-12i| = |-10| = 10$

(9) a) diverges (compare to $\sum \frac{1}{k}$)

b) geometric, $r = \frac{1}{2^3}$, sum $= \frac{1}{2^3} \cdot \frac{1}{1-\frac{1}{2^3}} = \frac{1}{8} \cdot \frac{1}{7/8} = 1/7$

c) geometric $= \frac{3^2}{1} + \frac{3^3}{4} + \frac{3^4}{4^2} + \dots = \frac{3^2}{1-3/4} = 9 \cdot 4 = 36$

d) converges (compare to $\sum \frac{1}{7^k}$)

e) $= \sum_{k=1}^{\infty} \frac{7^k}{21^k} - \sum_{k=1}^{\infty} \frac{3^k}{21^k} = \sum_{k=1}^{\infty} \frac{1}{3^k} - \sum_{k=1}^{\infty} \frac{1}{7^k}$

$= \frac{\frac{1}{3}}{1-1/3} - \frac{\frac{1}{7}}{1-1/7} = \frac{1}{2} - \frac{1}{6} = 1/3$

f) ~~1~~ $i-1-i+1+i-1-i+1+\dots$ diverges ($r = i$, $|r| = 1$)

g) $= \frac{2-2i}{3} + \left(\frac{1}{3} + \frac{1+i}{2} \cdot \frac{1}{3} + \left(\frac{1+i}{2}\right)^2 \cdot \frac{1}{3} + \dots\right)$

$= \frac{2-2i}{3} + \frac{\frac{1}{3}}{1-\frac{1+i}{2}} = 1 - \frac{1}{3}i$