Math 6B(1) Homework #1
Due in class Friday, Sept. 5.

For #1-4, find the Taylor polynomials of degree n approximating the given functions for x near 0.

1. $\dfrac{1}{1+x}$, $n = 4, 6, 8$
2. $\sqrt{1+x}$, $n = 2, 3, 4$
3. $\arctan x$, $n = 3, 4$
4. $\sqrt[3]{1-x}$, $n = 2, 3, 4$

For #5-6, find the Taylor polynomial of degree n for x near the given point a.

5. $\sin x$, $a = \pi/2$, $n = 4$
6. e^x, $a = 1$, $n = 4$
7. Suppose that g is a function which has continuous derivatives, and that $g(5) = 3$, $g'(5) = -2$, $g''(5) = 1$, and $g'''(5) = -3$.
   (a) What is the Taylor polynomial of degree 2 for g near 5? What is the Taylor polynomial of degree 3 for g near 5?
   (b) Use the two polynomials that you found in part (a) to approximate $g(4.9)$.
8. (a) Find the Taylor polynomial approximation of degree 4 about $x = 0$ for the function $f(x) = e^{x^2}$.
   (b) Compare this result to the Taylor polynomial approximation of degree 2 for the function $f(x) = e^x$ about $x = 0$. What do you notice?
   (c) Use your observation in part (b) to write out the Taylor polynomial approximation of degree 10 for the function in part (a).
   (d) What is the Taylor polynomial approximation of degree 5 for the function $f(x) = e^{-2x}$?

For #9-16, determine whether the sequence has a limit. Find the limit when it exists.

9. $1, 2/3, (2/3)^2, \ldots, (2/3)^{n-1}, \ldots$
10. $\cos\pi, \cos 2\pi, \cos 3\pi, \ldots, \cos n\pi, \ldots$
11. $2.9, 2.99, 2.999, \ldots$ (The nth term has n nines after the decimal point.)
12. $-2, 4, -8, 16, \ldots, (-2)^n, \ldots$
13. $\cos 1, \cos\frac{1}{2}, \cos\frac{1}{3}, \ldots, \cos\frac{1}{n}, \ldots$
14. $\{(0.98)^n)\}$
15. $\{(1.02)^n)\}$
16. $\dfrac{\sin 1}{1}, \dfrac{\sin 2}{2}, \dfrac{\sin 3}{3}, \ldots, \dfrac{\sin n}{n}, \ldots$
17. Some people think that the terms of a sequence without a limit necessarily increase or decrease without bound. Give an example disproving this idea.