Math 6B(1) Practice Midterm (not to be turned in)

1. Determine if the following series converge or diverge. Be sure to give reasons!

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2n^4 - 6n^3 + 13n}{n^5 + n^2 + 4}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^{n-1} + 1}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{(n!)(n!)}{(2n)!}$

(d) $\displaystyle\sum_{n=1}^{\infty} \left(1 + \frac{2}{n}\right)^n$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{2^n}$

(f) $\displaystyle\sum_{n=1}^{\infty} a_n$, if the nth partial sum of this series is given by $s_n = \dfrac{n-1}{2n+1}$.

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n} a_n$, if you know that $\displaystyle\sum_{n=1}^{\infty} a_n$ is a positive series that converges.

2. Find positive numbers A and B such that

$$0 < A \leq \sum_{n=0}^{\infty} \frac{1}{5^n + n^3} \leq B.$$

3. Suppose that at the beginning of each hour, a patient is given an injection of a 200 mg dose of antibiotics. It is known that after one hours, 43% of this antibiotic leaves one's system. So, the total amount of the drug *in* the patient after one hour is 300 + 300(.57) mg.

(a) How many mg of the drug are in the body after 24 hours?

(b) It turns out that 700 mg is a lethal dose of this antibiotic. Will the patient ever have this much of the drug in his/her system?

4. For each part, first determine whether or not such a series exists. If one does exist, give an example. If one does not exist, explain why not.

(a) A geometric series that converges to 2.

(b) A divergent series whose terms go to 0.

(c) A convergent series whose terms do not go to 0.

(d) A convergent series whose terms go to 0.

(e) A series that converges, but not absolutely.

5. Let $z = -3 + 2i$. Find $|z|$, z^2, and $1/z$.

6. Handout, # 4.2.2(b,c), p. 56.

7. Handout, # 4.3.1, p. 59.

8. Find the third-degree Taylor polynomial at the point $a = 1$ for the function $f(x) = \sqrt{x}$.

9. The *present value*, \$$P$, of a future payment, \$$B$, is the amount that would have to be deposited in a bank account today to produce exactly \$$B$ in the account at the relevant time in the future. With an interest rate of r, compounded annually, and a time period of t years, a deposit of \$$P$ grows to a future balance of \$$B$, where

$$B = P(1+r)^t, \text{ or equivalently, } P = \frac{B}{(1+r)^t}.$$

(So if my uncle promises to give me \$1000 in two years, and the interest rate is 5%, that's worth $\frac{\$1000}{(1.05)^2} = \$907.03$ to me now, because if I deposited \$907.03 today, I'd have \$1000 in two years.)

One way of valuing a company is to calculate the present value of all its future earnings. Suppose a farm expects to sell \$1000 worth of Christmas trees once a year forever, with the first sale in the immediate future. What is the present value of this Christmas tree business? Assume that the interest rate is 6% per year, compounded annually.