MATH 6B May 2003

Practice Final

Here are a bunch of practice problems, mostly stolen off of other people's old exams. Obviously, your exam will be shorter than this. (The review sections at the end of the chapters in Hughes-Hallett and Gillett are another good source of practice problems.)

1. Find the intervals of convergence (including endpoints!) for the following power series:

a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{n^2}$

b) $\displaystyle\sum_{n=0}^{\infty} nx^n$

c) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-7)^{2n}}{4^n}$

2. Determine if the following series converge or diverge.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^n}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n!n}$

c) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n}{2n+1}\right)$

d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^3-1}}$

e) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(n-3)^2+1}$

3. The series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges (how do you know?) to a sum S. How many terms would you need for a partial sum to approximate S to within 10^{-4}?

4. Find a power series solution of the form $\sum c_n x^n$ for the differential equation $\frac{dy}{dx} + y = 1$ with $y(0) = 2$.

5. The rhinoceros is now extremely rare. Suppose that enough game preserve land is set aside so that there is no danger of overcrowding. However, if the population is too small, fertile adults have difficulty finding each other when it is time to mate. Write a differential equation that models the rhinoceros population based on these assumptions. (Note that there is more than one reasonable model that fits these assumptions.)