

**MORE ERRATA IN EDWARD BELTRAMI, *MATHEMATICAL  
MODELS FOR SOCIETY AND BIOLOGY***

**3, 19:** Should be “ $2 \leq i \leq N - 1$ ” (Paul Blain)

**3, Figure 1.1:**  $P$ 's should be  $p$ 's. (Paul Blain)

**6, 18:** Add  $\square$  at the end of the line. (David Lovit)

**6, 20:** Remove  $\square$ . (David Lovit)

**8, 5:** “...finite (**nonzero**) number of moves...” (David Lovit)

**21–24:** There is a serious problem with the first example in this section. It arises because we're dealing with  $r_i$ , the number of groups off on day  $i$ , instead of what we're really interested in, namely  $n_i$ , the number of groups working on day  $i$ . This is okay as long as  $N$ , the number of groups, stays constant. However, when we introduce six-day workweeks, on the bottom of p. 23,  $N$  changes (a longer workweek requires fewer workers). Consider the example where  $n_1 = n_2 = \dots = n_7 = 5$  and  $N = 7$ , so that  $r_1 = r_2 = \dots = r_7 = 2$ . Then both  $x_1 = x_2 = \dots = x_7 = 1, x_8 = x_9 = \dots = x_{14} = 0$  and  $x_1 = x_2 = \dots = x_7 = 0, x_8 = x_9 = \dots = x_{14} = 2$  are solutions to system 2.2, but only the first one makes any sense. (In the second, we do have two groups off each day, but twelve groups working, far more than are needed.) An obvious way to solve this issue is to forget about the  $r_i$ 's and deal only with the more natural  $n_i$ 's.

The second example (beginning on the bottom of p. 24) does work out, because we assume that the number of four-day workweeks is the same as the number of six-day workweeks, so the average workweek is five days and the total number of workers needed stays constant regardless of the schedule picked.

**26, -4:** “Exercise 2.5.3” (David Lovit)

**28, 6:** “Exercise 2.5.2” (David Lovit)

**29, 14:** “the  $i$ th state the integer part...”

**31, 16:** Assuming that “round off to the nearest integer” means rounding fractional parts of .5 up, we can strengthen (2.11) by replacing the first “ $\leq$ ” with “ $<$ .” (Disha Katharani and White Srivisal) Also, it should be made explicit that the second inequality of (2.11) applies only to states with  $a_i > 1$ . (White Srivisal)

**36, 12:** **Erratum in the first list of errata!**  $x_j \leq r_j + 1$  should be  $x_j \leq r_{j+1}$  (David Lovit)

**36, 14:** Should be  $x_7 \leq \min(r_1, r_7)$ . (David Lovit and Rachel Thomas)

**36, -1:** By “maximizing the number of three-day weekends,” he means “maximizing  $x_7 + x_8$ .” (There are other ways to get three-day weekends, which we're ignoring in this problem). (Dan Keys)

**37, ex. 2.5.5:** Throughout,  $N$  is used to denote both a set (the set of all customers) and an integer (the total number of customers). (David Lovit)

**38, 8:** The sentence beginning “Define  $x_j$  to be...” should read “Define  $x_j$  to be the number of police working during the  $j$ th work period.” (David Lovit)

**67, 15:** (Proof of Theorem 4.1) There is a hole in the proof. For example, consider the graph



If we start at  $v_1 = v$ , go to  $w$ , then return to  $v_2 = v$ , then we’re not done, but we can’t “begin anew starting now at  $v_2$ ” since there are no unused edges starting from  $v$ . To fix the proof, use the fact that somewhere on the closed loop that we’ve created is a vertex with an unused edge.

**69, 9:** “roundtrip tour” – I don’t think that there’s any reason that a street sweeping path has to end where it started. So throughout this example, we should be talking about finding Euler paths, not Euler tours.

**70, Figures 4.4 & 4.5:** These figures don’t quite match - in Figure 4.5, the street from 1 to 4 is one-way, but in Figure 4.4 it’s not. (Disha Katharani)

**72, -5:** “Note that... Euler tour can be constructed.” The polarity condition is necessary, but not sufficient, for the existence of an Euler tour. We also need to verify that the resulting graph is connected.

**72, -4:** “an Euler tour”

**79, -1:** Should be “ $\mathbf{r} \cdot \mathbf{r}$ ”, not “ $r \cdot r$ .” (Long Si Ken)

**89, -11:** “Stokes’ theorem,” not “Stoke’s theorem.” (David Lovit)

**90, 11:** Should be “ $\text{Tw}(C_1, C_2) - \text{Tw}(C_2, C_1) = \text{Wr}(C_1) - \text{Wr}(C_2)$ .”

**90, 16:** Should be “ $\text{Wr}(C_2)$  is approximately  $n(1 - (1 + 4\pi^2 n^2 r^2 / L^2)^{-1/2})$ .”

**102, Lemmas 6.3 & 6.4:** These lemmas apply only in dimension two.

**118, Ex. 6.7.4:** There is some confusion with this problem and its solution.

The solution is for the assumption that one side loses troops at a rate proportional to the number of troops on the other side, not what is stated in the problem. In fact, using the assumption stated in the problem, one force or the other is eliminated asymptotically, *not* in finite time. (Dan Keys)

**118, Ex. 6.7.5:** Lemmas 6.2 and 6.3 don’t actually say anything about oscillation. (David Lovit)

**119, 2:** Should be “ $S + I = 1$  at  $t = 0$ .” (David Lovit)

**119, 4:** Should be “ $\frac{dI}{dS} =$ ”

**141, 3:** “litter” (Long Si Ken)

**144, 6:** “... as  $\Delta t \rightarrow 0$ ...”

**155, 5:**  $a, l, b > \mathbf{0}$  (David Lovit)

**155, 8:** “...  $(u, v) = (0, 0), (0, l)$  and  $(l, 0) \dots$ ” (Disha Katharani)

**159, -3 to -2:** “probabilities  $p_{i,i+1} = p$  and  $p_{i,i-1} = q$  for  $1 \leq i \leq N - 1$ ,  $p_{0,0} = p_{N,N} = 1$ , and  $p_{i,j} = 0$  for all other  $i, j$ .” (David Lovit)

**159, -1:** “Let  $b_i$  be the probability” (David Lovit)

**160, 1:** “(this is  $b_{i,N} \dots$ )” (David Lovit)

**160, 5:** Note that (8.30) holds only for  $1 \leq i \leq N - 1$ . (Dan Keys)

**160, 7:** “Now  $b_i$  is identical...” (David Lovit)

**160, 8:** “... =  $q(b_i - b_{i-1})$ ” (David Lovit)

**160, 11:**  $i = 1, 2, \dots, N$ . (Dan Keys)

**160, -13:** Exercise 1.5.3 (Disha Katharani, Dan Keys, Long Si Ken)

- 168, 14:**  $g(x)$  is not a probability density function, it's just nonnegative.  
 (This is reassuring, since (9.7) doesn't give a probability density function, in general.)
- 169, 16:** The “ $g(g)$ ” in the exponent should be “ $g(x)$ .”
- 169, 19:**  $a(x)p(x) - rb(x) \leq 0$
- 170, 8:** Exercise 9.6.1 (David Lovit)
- 171, 4:** “below” should be “above” (Dan Keys)
- 171, 7:** “above” should be “below” (Dan Keys)
- 189, 4:** “1.5.4” should be “1.5.5.”
- 189, 5:** “1.5.8” should be “1.5.7.” (Anton Voinov)
- 190, 8:** Should be  $\begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ . (Ian Sulam and Anton Voinov)
- 191, 19:** “4.5.4” should be “4.5.5.”
- 191, -10 and -9:** Should be “ $\mathbf{r}_1(\theta)$ ,” not “ $\mathbf{r}_1(s)$ ,” and  $\mathbf{r}_1$  and  $\mathbf{r}_2$  are reversed.  
 (Anton Voinov and Long Si Ken)
- 191, -7:**  $\gamma = 1 + \tau^2$ , not  $(1 + \tau^2)^2$ . (Adam Gerber)
- 194, 8.6.3:** Throughout, he is assuming that  $l = 1$ . (Disha Katharani)
- 194, -2:**  $\dots \int c(x) dx \leq \int p(x) dx \dots$  (David Lovit)