Math 61 Modeling Wiseman

Due in class Tuesday, 9/24/02

Your classmates are your target audience for all the write-ups.

(1) Come up with your own models, one involving scheduling and one involving apportionment. What questions are you trying to answer? How do your models help you answer them? What assumptions are you making? How would you improve your models?

(2) Suppose that you are faced with the problem of how to adjust traffic signals for rush hour traffic. What is the best way to do it? We suppose that at time $t = 0$ there is no line at the signal in either direction. At the end of the problem, try to decide how important and how realistic this assumption is. Cars arrive at the signals at rate $q_N(t)$ from the north and $q_E(t)$ from the east. The signal can handle cars at a rate k. Let $Q_N(t)$ and $Q_E(t)$ be the integrals of q_N and q_E from 0 to t.

(a) Show that T, the earliest possible time the intersection can be cleared, is determined by the equation $Q_N(T) + Q_E(T) = kT$.

(b) Let $f_N(t)$ be the flow of the north cars through the intersection at time t. Define F_N, f_E, and F_E in the obvious fashion. What relationships can you discover among the four functions just defined? Interpret the area between the curves Q_N and F_N in terms of delay time.

(c) Show that the total delay time at the intersection is a minimum if and only if both intersections are cleared simultaneously at time T. Determine T.

(d) What is the best form for F_N? Suppose that the rush hour traffic starts to arrive earlier from the north so that $q_N(t)$ is large when t is small but $q_E(t)$ is small when t is small.

(e) Discuss improvements and generalizations for the model. Among the problem that you could consider are flows from all four directions, lost time when signals change, and unequal rates of flow (the parameter k) in different directions.

(From Bender, *An Introduction to Mathematical Modeling.*)

(3) Consider the running-through-the-rain problem again. This time, model the person as a surface and the rain as a vector field. How is this an improvement over the models in the handouts? How is it worse? How would you use this new model to answer the question of how fast to run? With these assumptions, is it possible that the path that will keep you driest is not a straight line? How would you find the driest path?

(4) After they finish the new science center, they're going to put in a new, 100ft-by-200ft corner parking lot. The provost has asked for your help in designing the layout; that is, to design how the lines are to be painted.

You realize that squeezing as many cars into the lot as possible leads to right-angle parking with the cars aligned side by side. However, inexperienced drivers have difficulty parking their cars this way, which can give rise to expensive insurance claims. To reduce the likelihood of damage to parked vehicles, the college might then have to hire expert drivers for valet parking. On the other hand, most drivers seem to have little difficulty in parking in one attempt if there is a large enough turning radius from the access lane. Of course, the wide the access lane, the fewer cars that can be accommodated in the lot.

What do you tell the provost? (From the 1987 Mathematics Contest in Modeling.)

(5) This isn't too much of a problem at Swarthmore, but at most schools they have a lot of large classes in poorly designed lecture halls. Can you help your less fortunate peers?

(a) What are some criteria to be considered in designing a large lecture hall?

(b) One criterion is legibility of material written on the boards. Construct a model of legibility as a function of the distance your seat is from the board and the angle at which you look at the board. What will the curves of constant legibility look like on a floor plan? How can you test this prediction? Try it. Does this suggest shaping the back of the hall differently from what is usually done? How?

(c) Pick another criterion and develop a model for it.

(From Bender.)