Math 61 Modeling Wiseman

Due in class Thursday, 10/3/02

Your classmates are your target audience for all the write-ups.

(1) Come up with your own model involving graph theory. What questions are you trying to answer? How does your model help you answer them? What assumptions are you making? How would you improve your model?

(2) Critique the article on lecture hall design. What assumptions are they making? Do you agree with their conclusions? Are there any questions that come up now that weren't relevant when the article was written? Any that came up then that aren't relevant now? How would you improve their lecture hall?

(3) Critique Beltrami's street sweeping model. What assumptions is he making? Do you agree with his conclusions? How would you improve his model?

(4) The provost has asked for your help again. This time, she's worried about long-term planning for new buildings for campus (dorms, academic buildings, office space, gyms, etc.).
   (a) What are some criteria to be considered?
   (b) Pick a criterion and develop a model for it. What data do you need? How would you get them?

(5) (From Roberts, *Discrete Mathematical Models.)*
   (a) Consider the following model for traffic flow. We construct an undirected graph whose vertices are locations in a city. Two vertices are adjacent (i.e., there's an edge between them) if there's a (two-way) street between them. To cut down on traffic, the city is thinking of making all of its streets one-way, and they've asked you whether it's possible. (Well, of course it's possible; you can just put a one-way sign on each street. What they really want to know is whether you can do it in such a way that you can still get from any location to any other.)
   Phrased another way, the question is whether, given a connected, undirected graph, it's always possible to make it directed (by giving each edge a direction) in such a way that the resulting digraph is totally connected. If it is possible, try to prove it, or give an algorithm for assigning the directions. If it's not always possible, give some counterexamples. Formulate a hypothesis about when it is possible, and try to prove it.
   (b) Assuming that we can find a one-way street assignment that works, our next question is about its efficiency. One possible idea is to measure efficiency of a one-way street assignment by calculating the *diameter* of the resulting strongly connected digraph, that is the maximum of $d(u, v)$ for all vertices u and v. ($d(u, v)$ is the length of, i.e., number of edges in, the shortest path from u to v.)
      (i) Show on several examples of your choice some efficient and inefficient one-way street assignments, if this notion of efficiency is used.
      (ii) Comment on the efficiency of the orientation which alternates one-way streets, if the original graph is a street grid of East-West avenues and North-South streets.
      (iii) Comment on whether diameter is a good measure of efficiency and offer some alternatives.
      (iv) Try to develop algorithms for finding efficient one-way street assignments for each notion of efficiency.