Math 61 Modeling Wiseman

Due in class Thursday, 9/12/02

Your classmates are your target audience for all the write-ups.

(1) What assumptions are the various raindrop experts making when they answer the question of whether it's better to walk or run in the rain? Do you think their assumptions affect their answers? If so, how would you improve their assumptions, and how would you modify their models to take the new assumptions into account?

(2) A photocopy machine is always in one of three states: working, broken and fixable, broken and unfixable. If it is working, there's a 69.9% chance that it will be working tomorrow and a 0.1% chance that it will be broken and unfixable tomorrow. If it is broken and fixable today, there is a 49.8% chance that it will be working tomorrow and a 0.2% chance that it will be unfixable tomorrow. Unfixable means unfixable, so if it's unfixable there's no chance that it will be anything but unfixable tomorrow.

  (a) Draw a state diagram for this scenario. Categorize the states as absorbing and nonabsorbing.
  (b) Formulate the transition matrix. Label the states so that the identity matrix is in the upper left-hand block.
  (c) Compute the fundamental matrix $T = (I - Q)^{-1}$ and interpret the results. How long will this machine last? How much of that time will it be working, and how much of that time will it be under repair (broken and fixable)?

(From Mooney and Swift, *A Course in Mathematical Modeling*.)

(3) A hungry lion and a mathematician are placed in a circular ring. Both have the same top speed. Does the lion ever catch the mathematician? Try to come up with a model to answer this question. What assumptions are you making? How could you improve your model? (I don't know whether you'll be able to answer the question.) HINT: This doesn't necessarily have anything to do with Markov chains.

(4) Consider the following psychology experiment: A naive subject is seated at the end of a row of seven pretrained confederates of the experimenter. Instructions are read aloud which lead the subject to believe that he is participating in the following experiment on visual perception: On each of a sequence of trials, the eight individuals present are required to choose aloud (and from a distance) that particular one from among three comparison lines which has the same length as a standard line.

For each set of lines, the naive subject's turn to choose comes only after he has heard the unanimous, though incorrect, responses of the seven confederates. Thus, he is motivated, on the one hand, to answer according to his perception and, on the other, to conform to the unanimous choice of the group.

One subject gave the following responses. (Choices were recorded as R if he gave the right answer, and W if he agreed with the others and gave the wrong answer. The first trial is on the left.)

$$RRRWWRRRWWWRRRRWRRRRRRRRRRRRRRRRRR$$

Can you come up with a Markov chain model that might explain these data? Don't spend too long on this question. (From Kemeny and Smell, *Mathematical Models in the Social Sciences*.)

(5) Make up your own Markov chain model. More specifically, come up with a problem that can be modeled with a Markov chain, draw a state diagram, and compute the transition matrix. What problems do you foresee in using this model to analyze the problem? (You don't have to do any of the actual analysis.)

(6) Consider a rental car company with branches in Orlando and Tampa. Each rents cars to Florida tourists. The company specializes in catering to travel agencies that want to arrange tourist activities in both Orlando and Tampa. Consequently, a traveler will rent a car in one city and drop it off in either city. This can lead to an imbalance in available cars to rent. Historical data on the percentage of cars rented and returned to these locations are reflected in the transition matrix below:

	Orlando	Tampa
Orlando	0.6	0.4
Tampa	0.3	0.7

(Thus 40% of the cars rented in Orlando are returned to Tampa, etc.) Assuming that all of the cars are originally in Orlando, figure out the long-term behavior of the percentages of cars available at each location. Try to solve this a couple different ways. What happens if you start with different initial conditions (that is, not all the cars start in Orlando)? (From Giordano, Weir, and Fox, *A First Course in Mathematical Modeling*.)