Math 50 Practice 1st Midterm Solutions

1 a) This is exact. It's the same as $(t+\sin(ty)+y^2)'=0$, so the solution is $t+\sin(ty)+y^2=C$, which gives y implicitly as a function of t.

b) This is separable. Either $y=0$, or $\ln(|y|) = t^2/2 + C$, so $y = De^{t^2/2}$.

c) This is linear, so we find an integrating factor μ. Taking $\mu(t)=e^{t^2}$, we find that $y = \frac{-1}{2} + Ce^{-t^2}$. Plugging in $y(0)=2$, we get $C=\frac{5}{2}$, so $y=\frac{-1}{2}+\frac{5}{2}e^{-t^2}$ is the soln.

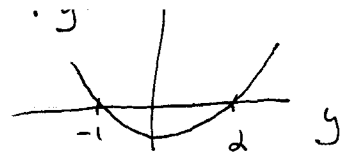
2) This is linear inhomogeneous. 1st, solve the assoc. homog. problem $y''+3y'+2y=0$. Ans: $y=C_1e^{-t}+C_2e^{-2t}$. Next, find a particular soln. Guess: $y = a\cos t + b\sin t$. Solving, we get $a=\frac{1}{10}$, $b=\frac{3}{10}$. So the general soln. is $y=\frac{1}{10}\cos t+\frac{3}{10}\sin t+C_1e^{-t}+C_2e^{-2t}$

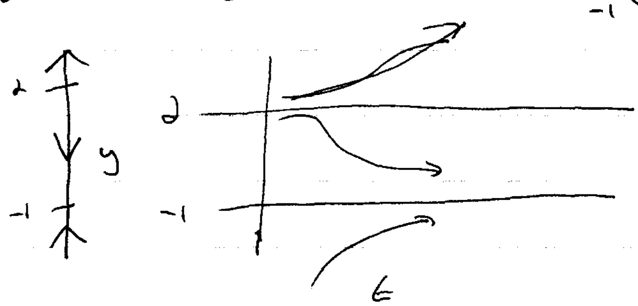
3) The solns to $y''-4y=0$ are $C_1e^{2t}+C_2e^{-2t}$, which tend to 0 iff $C_1=0$. Solving $y(0)=y_0$ and $y'(0)=y_0'$ for C_1, we get $C_1=\frac{1}{4}(y_0'+2y_0)$, which is 0 if $y_0'=-2y_0$. The solns. for all other initial conditions tend to either $+\infty$ or $-\infty$.

4) a) 3rd order, nonautonomous, nonlinear b) 2nd order, autonomous, linear, homogeneous
c) 1st order, autonomous, linear, homogeneous

5) It looks like all the solutions fail to exist forever in backward time. If $y_0=-2$, solns. are not unique.

6) $y'=0$ when $y=-1$ or 2:





$y=-1$ is a stable equilibrium, $y=2$ is unstable. Thus solns. beginning above 2 tend toward $+\infty$, those beginning below -1 tend toward -1, and those b/w -1 & 2 tend toward -1.

7) Need to show that t & $t\ln(t)$ are solns (plug in and check), and that the Wronskian is nonzero. $W(t)=\begin{vmatrix} t & t\ln(t) \\ 1 & \ln t + 1 \end{vmatrix} = t$, which is $\neq 0$ on $(0,\infty)$, so they do form a fund. set. Set $y=C_1 t + C_2 t\ln(t)$ and plug in $y(1)=-1$, $y'(1)=2$ to get $y=-t+3t\ln(t)$.

8) This is separable. Either $y=0$, or $\frac{1}{y}\,dy = \frac{1+t}{t}\,dt \Rightarrow \ln|y| = \ln|t| + t + C$ $\Rightarrow y = \pm t e^{t} e^{C}$, so $y = D t e^{t}$, where D can be any real number. There are infinitely many solns. with $y(0)=0$, since $y(0)=0$ for any choice of D. The theorem does not apply, since in the eqn. $y' = \frac{1+t}{t} y$, $\frac{1+t}{t}$ is not continuous at $t=0$.

9) Let $A(t)$ = mg in blood at time t. It decays exponentially, so $A(t)=Ce^{rt}$ ($A(0)=C$, so C is the initial dose). The half-life is 5 hrs, so $\frac{1}{2}C = Ce^{r\cdot 5} \Rightarrow r = -\frac{1}{5}\ln 2$. We want 45.50 mg in the blood at time 1 hr, so $A(1) = Ce^{-\frac{1}{5}(\ln 2)t} = 45.50 \Rightarrow C = 45.50\cdot\sqrt[5]{2}$.

10) See section 2.4.