Math 30 February 26, 2002 Name:_____

FIRST MIDTERM

This exam is 7 pages long; check that you have all the pages. Show your work. Correct answers with no justification may receive little or no credit. No calculators, notes, or books are allowed. No uncalled-for simplification is required. Use the backs of pages if you run out of space, and make sure that I can find your answers.

THINK JOYFULLY

PROBLEM	POINTS	SCORE
1	20	
2	15	
3	10	
4	20	
5	10	
6	15	
7	10	
Extra credit	2	
TOTAL	100	

(1) (20 pts) Solve the following differential equations. (Give the general solution if no initial condition is specified.)

(a) $t^5 y' + y^5 = 0$

(b) $e^{2y} - y\cos(ty) + (2te^{2y} - t\cos(ty) + 2y)y' = 0$

(c) $\dfrac{dy}{dx} + 2xy = x$, $y(0) = -3$

(2) (15 pts) Find the general solution to the ODE $y'' - 2y' + y = e^{-t}$.

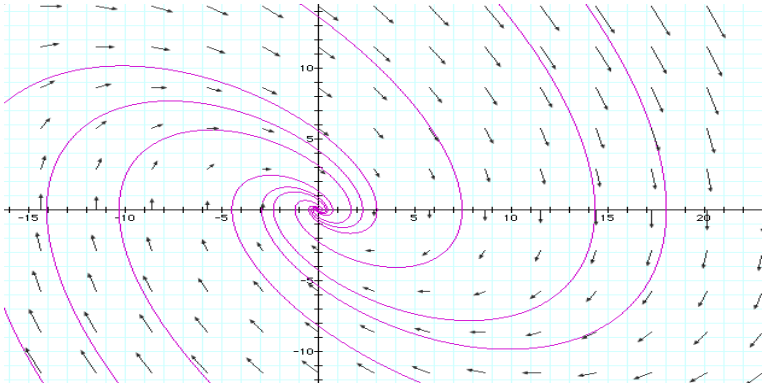
(3) (10 pts) Consider the initial value problem $y'' - 3y' + 2y = 0$, $y(0) = y_0$, $y'(0) = y_0'$. For what initial conditions (i.e., what values of y_0 and y_0') will the solution tend to 0 as $t \to \infty$?

(4) (20 pts) Newton's law of cooling states that the rate at which the temperature $T(t)$ changes in a cooling body is proportional to the difference between the temperature in the body and the constant temperature T_m of the surrounding medium. That is, $\dfrac{dT}{dt} = k(T - T_m)$.
Yesterday, when I took my delicious cake out of the oven, I measured its temperature to be 300°F. Three minutes later its temperature was 200°F. How long will it take to cool off to 80°F if the room temperature is a balmy 70°F?

(5) (10 pts) Discuss the differences between linear homogeneous and linear inhomogeneous ODEs.

(6) (15 pts) Find the equilibria for the ODE $y' = -y^2 - y$. Are they stable, unstable, or semistable? What is the long-term behavior of the solutions?

(7) (10 pts) The figure below shows the vector field $\frac{d}{dt}\left(\begin{bmatrix} y \\ y' \end{bmatrix}\right)$ associated to a differential equation $y'' = F(y, y')$, along with some solution curves. Discuss the long-term behavior of solutions to the differential equation.



EXTRA CREDIT (2 points) Would you vote for Senator Haddock? Why or why not?