Math 16(1) Practice Midterm (not to be turned in)

1. A. Find the solution set to the following system of equations:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &= 15 \\ 4x_1 + 5x_2 + 6x_3 + 7x_4 &= 6 \\ 6x_1 + 7x_2 + 8x_3 + 9x_4 &= 0 \end{aligned}$$

B. Consider the system of equations:

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 0 \\ -3x_1 - x_2 + 2x_3 &= 0 \\ 5x_2 + 3x_3 &= 0 \end{aligned}$$

Asking if this has a unique solution is equivalent to which of the following?

1. The vectors $\begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ are linearly independent.
2. IGNORE
3. The equation $\begin{bmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \\ 0 & 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ has a solution for every choice of b_1, b_2, b_3.
4. $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \in \text{Span}\left\{ \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$.
5. The transformation $\mathbf{x} \mapsto \begin{bmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \\ 0 & 5 & 3 \end{bmatrix} \mathbf{x}$ is one-to-one.
6. The rows of $\begin{bmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \\ 0 & 5 & 3 \end{bmatrix}$ are linearly independent.
7. The range (image) of the transformation that sends $\mathbf{x}$ to $\begin{bmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \\ 0 & 5 & 3 \end{bmatrix} \mathbf{x}$ is a subset of $\mathbb{R}^3$.

C. Find the inverse of the coefficient matrix in part B and use it to solve the matrix equation

$$\begin{bmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \\ 0 & 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

2. A. Is the set of vectors $\begin{bmatrix} 0 \\ 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -5 \\ -7 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 10 \\ 4 \\ -4 \\ 2 \end{bmatrix}$ linearly independent?

B. Prove the following theorem from our book: Let $\{\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_p}\}$ be a set of vectors from $\mathbb{R}^n$. If $p > n$, then this set is linearly dependent.

3. A. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation that multiplies a vector by the scalar 2 and then reflects it across the x-axis. IGNORE THE REST OF PART A.

B. Find the standard matrix for T.

C. Is T one-to-one and onto? Justify your answer, of course.

D. IGNORE.

4. A. Let A be an $n \times n$ matrix whose columns are linearly independent. Show that the columns of A^2 span $\mathbb{R}^n$.

B. How many pivot columns must a 7×5 matrix have if its columns are linearly independent? Why?

C. IGNORE.

5. Are the following true or false?

- If the system $A\mathbf{x} = \mathbf{b}$ has more than one solution, then so does the system $A\mathbf{x} = \mathbf{0}$.
- If A is a 6×5 matrix, then the linear transformation that send $\mathbf{x}$ to $A\mathbf{x}$ cannot map $\mathbb{R}^5$ *onto* $\mathbb{R}^6$.
- If A and B are $m \times n$, then both AB^T and A^TB are defined.
- If A and B are $n \times n$, then $(A + B)(A - B) = A^2 - B^2$.
- IGNORE.