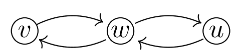
ERRATA IN EDWARD BELTRAMI, *MATHEMATICAL MODELS FOR SOCIETY AND BIOLOGY*

3, 7: Clearer: "… $p_{i,j}$ denotes the observed fraction, out of all the moves from state i, of those that go to state j."
3, 19: Should be "$2 \leq i \leq N-1$"
3, Figure 1.1: P's should be p's.
4, -1: Remove $\square$.
5, 4: Add $\square$ at the end of the line.
5, 17: "Transient" is defined in the next paragraph.
6, 16: (Proof of Lemma 1.2) "With probability one, each transient state will be visited only a finite number of times, as we…"
6, 18: Add $\square$ at the end of the line.
6, 20: Remove $\square$.
6, 21: (Theorem 1.1) This **I** is $N-s$ by $N-s$.
6, -12: Remove $\square$.
6, -7: Add $\square$ at the end of the line.
8, 5: "… finite **(nonzero)** number of moves…"
10, 3: "… and $(\mathbf{I}-\mathbf{Q})^{\mathbf{-1}}$ …"
11, 8: (Formula 1.7) $M_i = b_{i,1} + \sum_{j=3}^{7} t_{i,j} - 1$
11, 9: "(Exercise 1.5.4…"
12, bottom: Figure 1.3 is missing numerical labels. 1 is "No more crime," 2 is "Commit a crime," 3 is "Get arrested," and 4 is "Convicted and incarcerated."
15, 4: Should be $h_{i,i}$, not $h_{i,1}$.
21–24: There is a serious problem with the first example in this section. It arises because we're dealing with r_i, the number of groups off on day i, instead of what we're really interested in, namely n_i, the number of groups working on day i. This is okay as long as N, the number of groups, stays constant. However, when we introduce six-day workweeks, on the bottom of p. 23, N changes (a longer workweek requires fewer workers). Consider the example where $n_1 = n_2 = \cdots = n_7 = 5$ and $N = 7$, so that $r_1 = r_2 = \cdots = r_7 = 2$. Then both $x_1 = x_2 = \cdots = x_7 = 1, x_8 = x_9 = \cdots = x_{14} = 0$ and $x_1 = x_2 = \cdots = x_7 = 0, x_8 = x_9 = \cdots = x_{14} = 2$ are solutions to system 2.2, but only the first one makes any sense. (In the second, we do have two groups off each day, but twelve groups working, far more than are needed.) An obvious way to solve this issue is to forget about the r_i's and deal only with the more natural n_i's.

The second example (beginning on the bottom of p. 24) does work out, because we assume that the number of four-day workweeks is the same as the number of six-day workweeks, so the average workweek is five days and the total number of workers needed stays constant regardless of the schedule picked.

26, -4: "Exercise 2.5.**3**"
28, 6: "Exercise 2.5.**2**"
29, 14: "the i**th** state the integer part. . . "
29, -11: Should be a_i, not a_2.
31, 16: Assuming that "round off to the nearest integer" means rounding fractional parts of .5 up, we can strengthen (2.11) by replacing the first "$\leq$" with "$<$." Also, it should be made explicit that the second inequality of (2.11) applies only to states with $a_i > 1$.
36, 9: $x_j = r_j - x_{j+7} - x_{j-1} \leq r_j - x_{j-1}$
36, 12: $x_j \leq r_{j+1}$
36, 14: Should be $x_7 \leq \min(r_1, r_7)$.
36, -9: "Otherwise $x_{j+\mathbf{7}}$ is nonzero. . . "
36, -1: By "maximizing the number of three-day weekends," he means "maximizing $x_7 + x_8$." (There are other ways to get three-day weekends, which we're ignoring in this problem).
37, ex. 2.5.5: Throughout, N is used to denote both a set (the set of all customers) and an integer (the total number of customers).
38, 8: The sentence beginning "Define x_j to be. . . " should read "Define x_j to be the number of police working during the jth work period."
44, 15: Add $\square$ at the end of the line.
46, 2: Add $\square$ at the end of the line.
46, 15: ". . . this constitutes m independent . . . "
46, -5: ". . . general hypotheses."
47, -14: $m = n\lambda h = t\lambda \ldots$
66: Should give a precise definition of what is meant by "graph," since different people use it to mean different things. Here, we seem to be allowing no edges from a node to itself (otherwise, Theorem 4.3 is not true).
67, 11: ". . . is called a**n** *Euler tour. . .* "
67, 13: Theorem 4.1 should read: *A connected directed graph has an Euler tour if and only if the difference between the inner and outer degrees is zero at each node.*
67, 15: (Proof of Theorem 4.1) There is a hole in the proof. For example, consider the graph



If we start at $v_1 = v$, go to w, then return to $v_2 = v$, then we're not done, but we can't "begin anew starting now at v_2" since there are no unused edges starting from v. To fix the proof, use the fact that somewhere on the closed loop that we've created is a vertex with an unused edge.
67, -12: "covers all the **edges** we are done. . . "
67, -4: ". . . not directed a**n** Euler tour. . . "
68, 1: I'm not sure what "the degree condition of Theorem 4.2.1" is (for one thing, there is no Theorem 4.2.1), but I believe the lemma is true for any graph.
69, 9: "roundtrip tour" – I don't think that there's any reason that a street sweeping path has to end where it started. So throughout this example, we should be talking about finding Euler paths, not Euler tours.
70, Figures 4.4 & 4.5: These figures don't quite match - in Figure 4.5, the street from 1 to 4 is one-way, but in Figure 4.4 it's not.

71, -8: "…supply nodes 4, 7, and **14**…"
72, -5: "Note that…Euler tour can be constructed." The polarity condition is necessary, but not sufficient, for the existence of an Euler tour. We also need to verify that the resulting graph is connected.
72, -4: "**an** Euler tour"
74, -3: "…Implicit in this **is** that each…"
79, -11: The picture in the plane, described in the first sentence of §5.2, is **not** shown in Figure 5.1.
79, -1: Should be "$\mathbf{r}\cdot\mathbf{r}$", not "$r\cdot r$."
80, Figure 5.1: The center point should be labeled p, not P.
80, 5: Should be dV, not dR.
81, -6: "…think of $C_{\mathbf{1}}$ as lying…"
82, figure 5.3: Points should be labeled p and p', not P and P'.
82, Figure 5.3 caption: "…surface S bound**ed** by one…"
82, 5-7: The situation described here, with two surfaces enclosing a region R, is not really pictured in Figure 5.3.
83, 9: Again, this is not quite what's shown in Figure 5.3.
87, 11: The second "cos" in formula (5.9) should not be in italics.
89, -11: "Stokes' theorem," not "Stoke's theorem."
90, 11: Should be "$\mathrm{Tw}(C_1, C_2) - \mathrm{Tw}(C_2, C_1) = \mathrm{Wr}(C_1) - \mathrm{Wr}(C_2)$."
90, 16: Should be "$\mathrm{Wr}(C_2)$ is approximately $n(1-(1+4\pi^2n^2r^2/L^2)^{-1/2})$."
102, Lemmas 6.3 & 6.4: These lemmas apply only in dimension two.
117, -5: Should read

$$\int \frac{x'(t)}{f(x(t))}\,dt = t + \text{constant}$$

117, -3: Should be $\int \frac{1}{f(x)}\,dx$, not $\int f(x)\,dx$.
118, Ex. 6.7.4: There is some confusion with this problem and its solution. The solution is for the assumption that one side loses troops at a rate proportional to the number of troops on the other side, not what is stated in the problem. In fact, using the assumption stated in the problem, one force or the other is eliminated asymptotically, *not* in finite time.
118, Ex. 6.7.5: Lemmas 6.2 and 6.3 don't actually say anything about oscillation.
119, 2: Should be "$S + I = 1$ at $t = 0$."
119, 4: Should be "$\dfrac{dI}{dS} =$."
124, -10 and -9: "…equation (7.3) has one, two, or three equilibria…"
130, 7: "…the waters a**n** impenetrable…"
136, 8: The second derivative is less than or equal to zero, **not** necessarily negative.
136, 9: Should be $\leq$, not $<$.
136, 11: Should be $\leq$, not $<$.
141, 3: "lit**t**er"
144, 6: "…as $\Delta t \to 0$…"
155, 5: $a, l, b > \mathbf{0}$
155, 8: "…$(u, v) = (0, 0), (0, l)$ and $(l, 0)$…"
159, -3 to -2: "probabilities $p_{\mathbf{i},i+1} = p$ and $p_{\mathbf{i},i-1} = q$ for $1 \leq i \leq N-1$, $p_{0,0} = p_{N,N} = 1$, and $p_{i,j} = 0$ for all other i, j."

159, -1: "Let $b_{\mathbf{i}}$ be the probability"
160, 1: "(this is $b_{\mathbf{i},N} \ldots$"
160, 5: Note that (8.30) holds only for $1 \leq i \leq N-1$.
160, 7: "Now $b_{\mathbf{i}}$ is identical..."
160, 8: "$\cdots = q(b_{\mathbf{i}} - b_{i-1})$"
160, 11: $i = 1, 2, \ldots, N$.
160, -13: Exercise 1.5.**3**
165, 6: "U-boat" usually refers to German submarines.
168, 14: $g(x)$ is not a probability density function, it's just nonnegative. (This is reassuring, since (9.7) doesn't give a probability density function, in general.)
169, 16: The "$g(g)$" in the exponent should be "$g(x)$."
169, 19: $a(x)p(x) - rb(x) \leq 0$
170, 8: Exercise 9.**6**.1
171, 4: "below" should be "above"
171, 7: "above" should be "below"
189, 4: "1.5.4" should be "1.5.5."
189, 5: "1.5.8" should be "1.5.7."
190, 8: Should be $\begin{pmatrix} 1&0&0&1 \\ 1&0&0&0 \\ 0&1&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&1&1 \end{pmatrix}$.
191, 19: "4.5.4" should be "4.5.5."
191, -10 and -9: Should be "$\mathbf{r}_1(\theta)$," not "$\mathbf{r}_1(s)$," and $\mathbf{r}_1$ and $\mathbf{r}_2$ are reversed.
191, -7: $\gamma = 1 + \tau^2$, not $(1+\tau^2)^2$.
194, 8.6.3: Throughout, he is assuming that $l = 1$.
194, -2: $\ldots \int c(x)\,dx \leq \int p(x)\,dx \ldots$