Math 325 Modeling

Due at the beginning of class Monday, February 6. Be prepared to present your answers.

(1) A photocopy machine is always in one of three states: working, broken and fixable, broken and unfixable. If it is working, there's a 69.9% chance that it will be working tomorrow and a 0.1% chance that it will be broken and unfixable tomorrow. If it is broken and fixable today, there is a 49.8% chance that it will be working tomorrow and a 0.2% chance that it will be unfixable tomorrow. Unfixable means unfixable, so if it's unfixable there's no chance that it will be anything but unfixable tomorrow.

(a) Draw a state diagram for this scenario. Categorize the states as absorbing and nonabsorbing.

(b) Formulate the transition matrix. Label the states so that the identity matrix is in the upper left-hand block.

(c) Compute the fundamental matrix $T = (I - Q)^{-1}$ and interpret the results. How long will this machine last? How much of that time will it be working, and how much of that time will it be under repair (broken and fixable)?

(From Mooney and Swift, *A Course in Mathematical Modeling.*)

(2) Consider the following psychology experiment: A naive subject is seated at the end of a row of seven pretrained confederates of the experimenter. Instructions are read aloud which lead the subject to believe that he is participating in the following experiment on visual perception: On each of a sequence of trials, the eight individuals present are required to choose aloud (and from a distance) that particular one from among three comparison lines which has the same length as a standard line.

For each set of lines, the naive subject's turn to choose comes only after he has heard the unanimous, though incorrect, responses of the seven confederates. Thus, he is motivated, on the one hand, to answer according to his perception and, on the other, to conform to the unanimous choice of the group.

One subject gave the following responses. (Choices were recorded as R if he gave the right answer, and W if he agreed with the others and gave the wrong answer. The first trial is on the left.)

$$RRRWWRRRWWWRRRRWRRRRRRRRRRRRRRRRRR$$

Can you come up with a Markov chain model that might explain these data? Don't spend too long on this question. (From Kemeny and Snell, *Mathematical Models in the Social Sciences.*)

(3) Make up your own Markov chain model. More specifically, come up with a system that can be modeled with a Markov chain, draw a state diagram, and compute the transition matrix. What issues do you foresee in using this model to analyze the problem? Pose a (reasonably) interesting question about your system, and use your mastery of Markov chain analysis to answer the question.