(1) Beltrami 2.5.2
(2) Beltrami 2.5.4
(3) Beltrami 2.5.6
(4) Say we have $s=3$ states and house size $h=5$. In 1990, state A has 51,000 inhabitants, state B 30,000, and state C 19,000 . What is the Hamilton apportionment?

In 2000 , the populations are $52,000,33,000$, and 15,000 , respectively. Now what's the Hamilton apportionment? Something horrible has happened here. What is it, and how do you feel about it?
(5) (Optional) Apportionment methods can also be characterized in terms of minimizing the distance between the quota point $q=\left(q_{1}, \ldots, q_{s}\right)$ and the integer apportionment point $a=\left(a_{1}, \ldots, a_{s}\right)$, where $\sum_{i=1}^{s} a_{i}=h=$ $\sum_{i=1}^{s} q_{i}$ for a house size of $h$ seats and $s$ states.
(a) Show that the apportionment scheme that minimizes (over all possible choices of $a$ ) the Euclidean distance

$$
d_{E}(a, q)=\left(\sum_{i=1}^{s}\left(a_{i}-q_{i}\right)^{2}\right)^{1 / 2}
$$

is the Hamilton method. (Recall that the Hamilton method works by giving each state its lower quota, i.e., the integer part $\left\lfloor q_{i}\right\rfloor$, and then dividing up the $k$ remaining seats among the $k$ states with the greatest fractional parts $q_{i}-\left\lfloor q_{i}\right\rfloor$.)
(b) Show that the apportionment scheme that minimizes the taxicab distance

$$
d_{T}(a, q)=\sum_{i=1}^{s}\left|a_{i}-q_{i}\right|
$$

is also the Hamilton method. Can you generalize?

